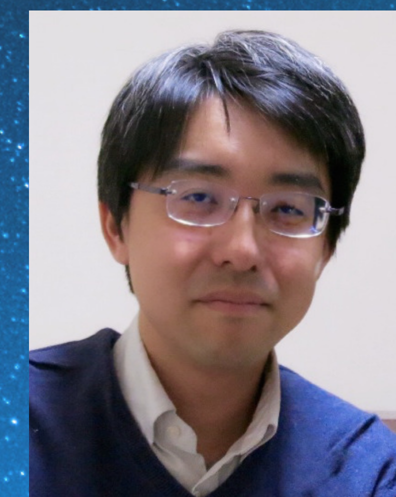
ITB Online Summer School on Galaxies dan Cosmology 2020

Introduction to weak gravitational lensing

Deciphering Dark Matter from Galaxies to the Universe



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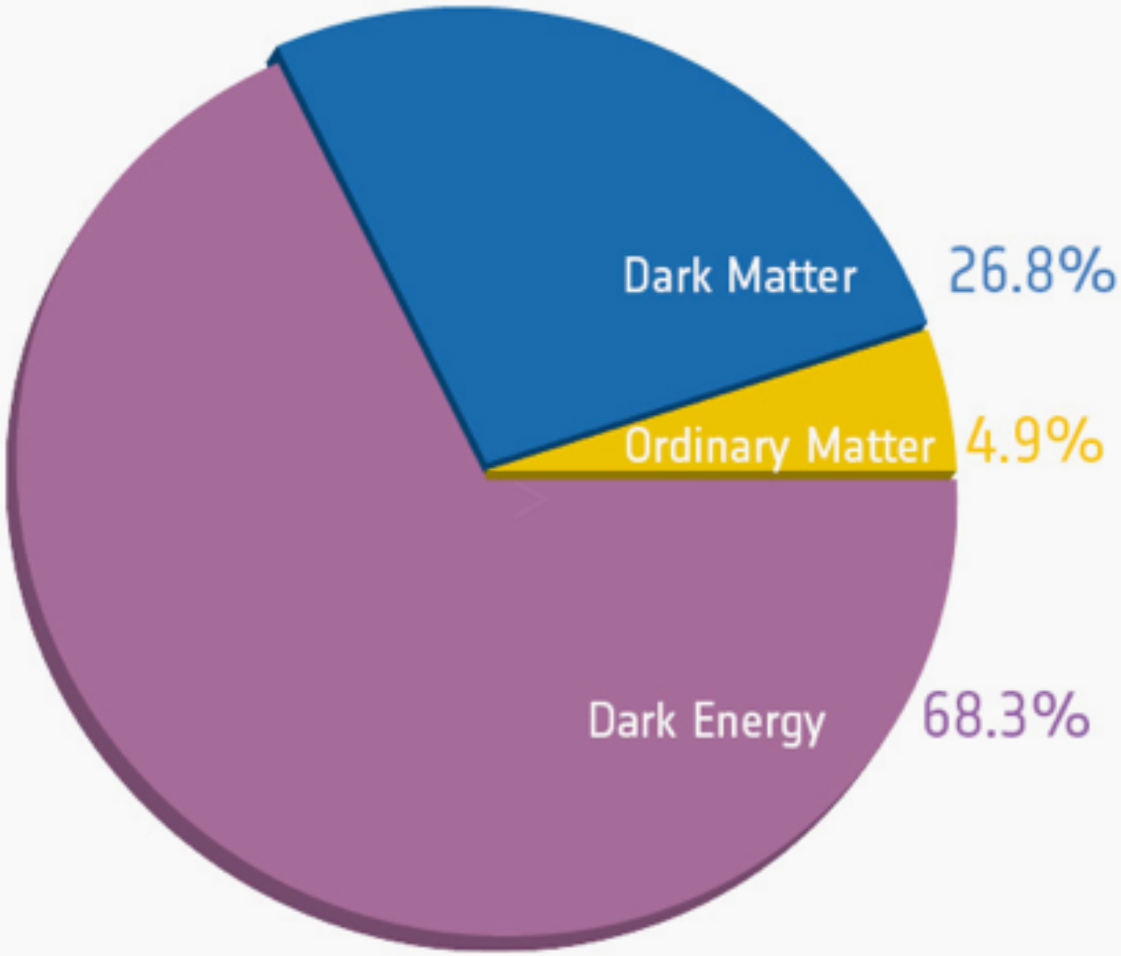
Plan of this lecture

- general introduction
- lens equation
- weak lensing shear and convergence
- tangential shear
- example of analysis
- weak lensing mass map

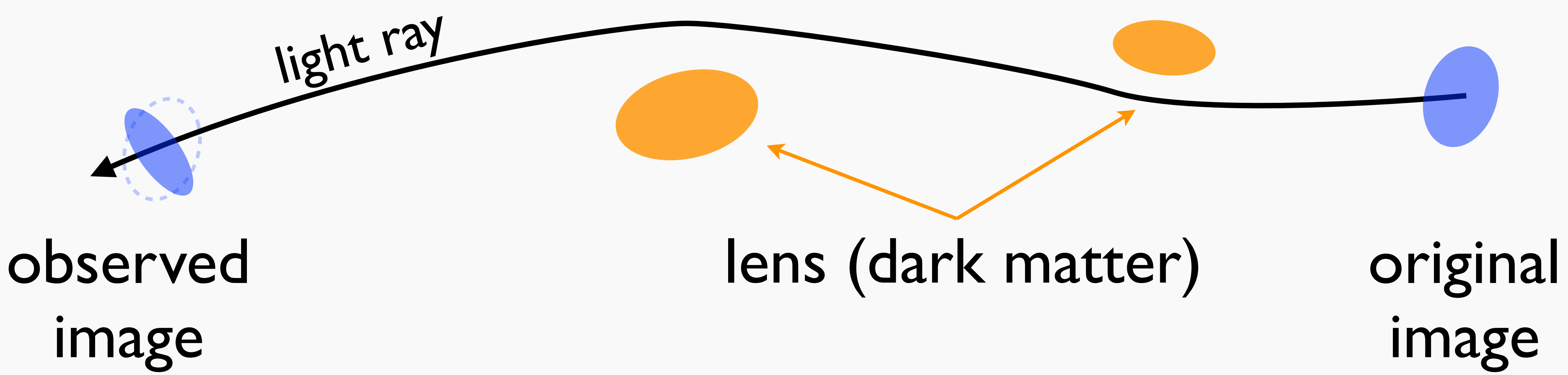
Standard cosmological model

- unknown components called **dark matter** and **dark energy**
- can explain many observations in a **consistent** manner



(ESA/Planck)

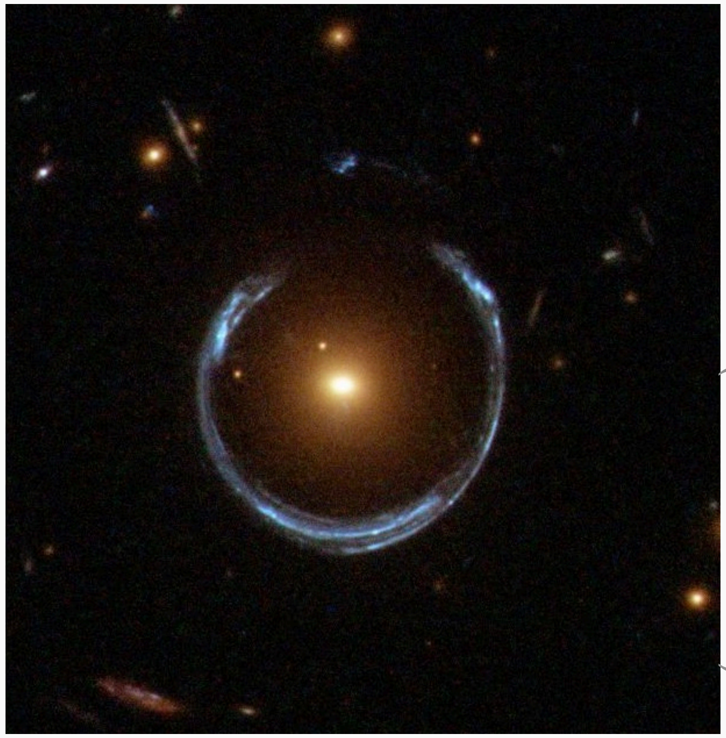
Gravitational lensing



- effect predicted by **general relativity**
- deflection of light ray due to intervening matter
- observed shapes **distorted**

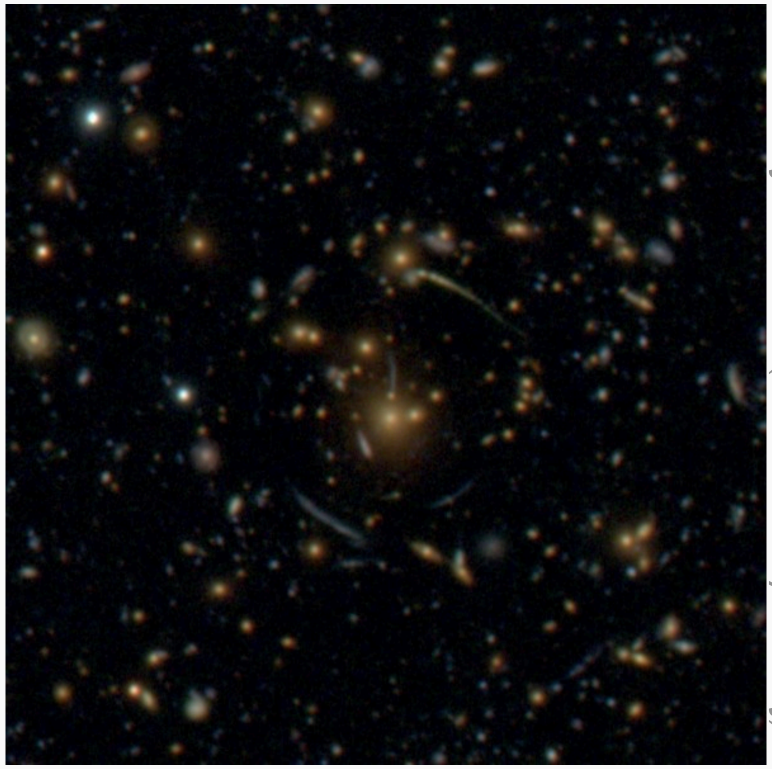
Observed gravitational lensing


A370 (Hubble/ESA/NASA)

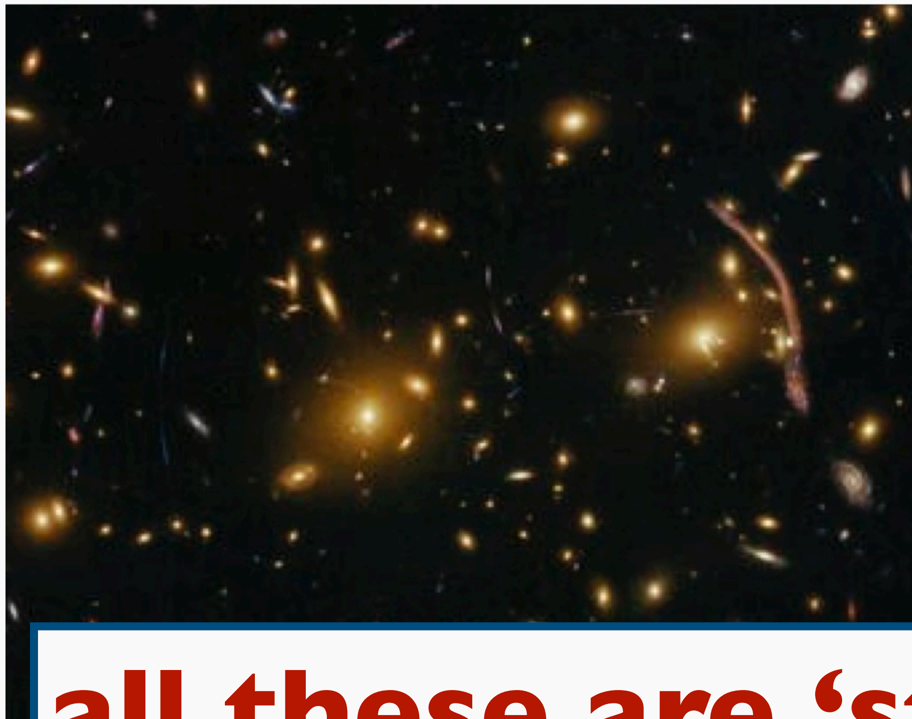

Cosmic Horseshoe (Hubble/ESA/NASA)


RCS2 032727-132623 (Hubble/ESA/NASA)


SDSS J1038+4849 (Hubble/ESA/NASA)


SDSS J1050+0017 (Subaru/U.Tokyo/NAOJ)

Observed gravitational lensing



A370 (Hubble/ESA/NASA)

Cosmic Horseshoe (Hubble/ESA/NA

RCS2 032727-132623 (Hubble/ESA/NASA)

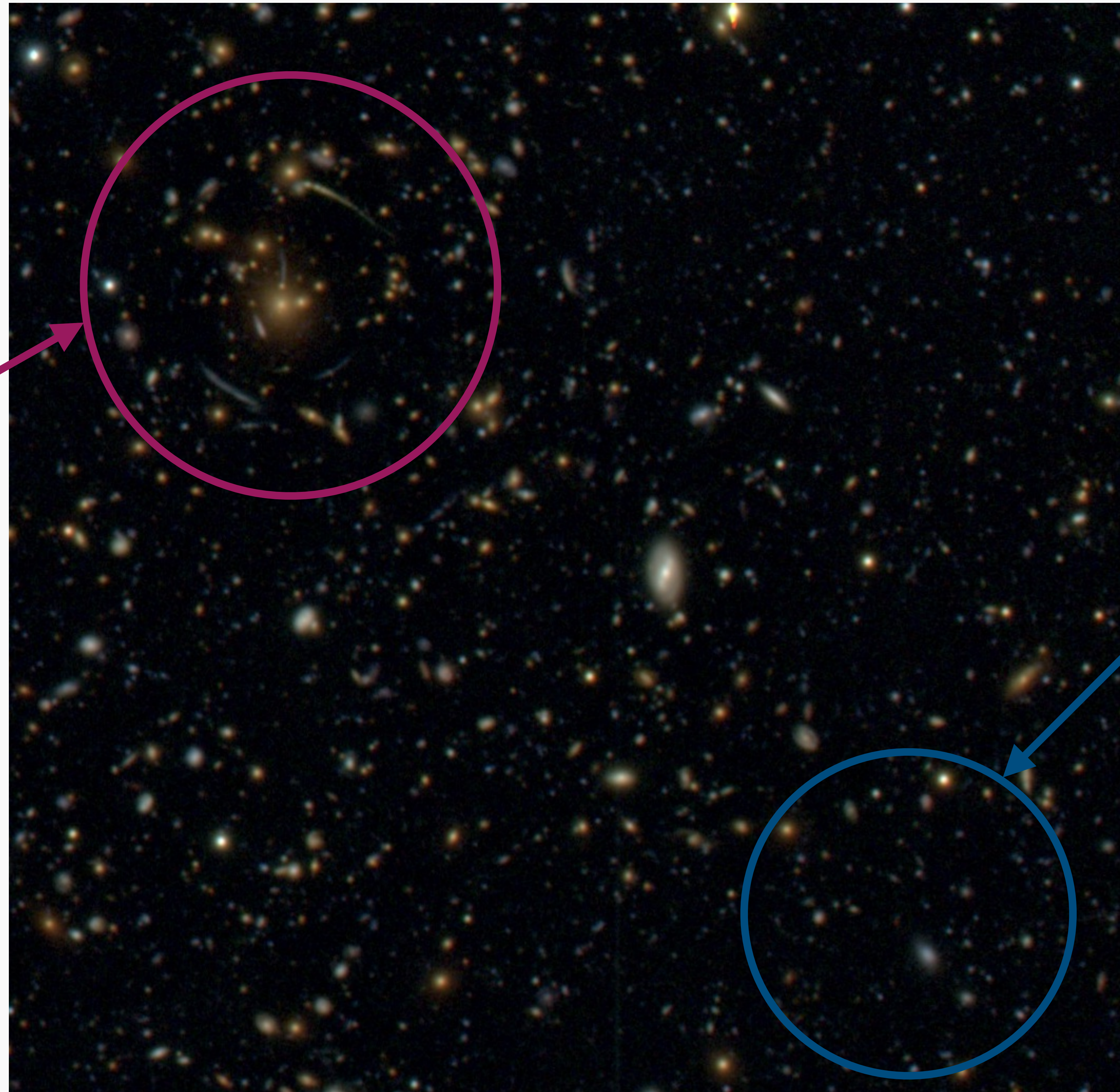
all these are 'strong' gravitational lensing!

SDSS J1038+4849 (Hubble/ESA/NASA)

SDSS J1050+0017 (Subaru/U.Tokyo/NAOJ)

Strong and weak lensing

strong lensing visible by eye

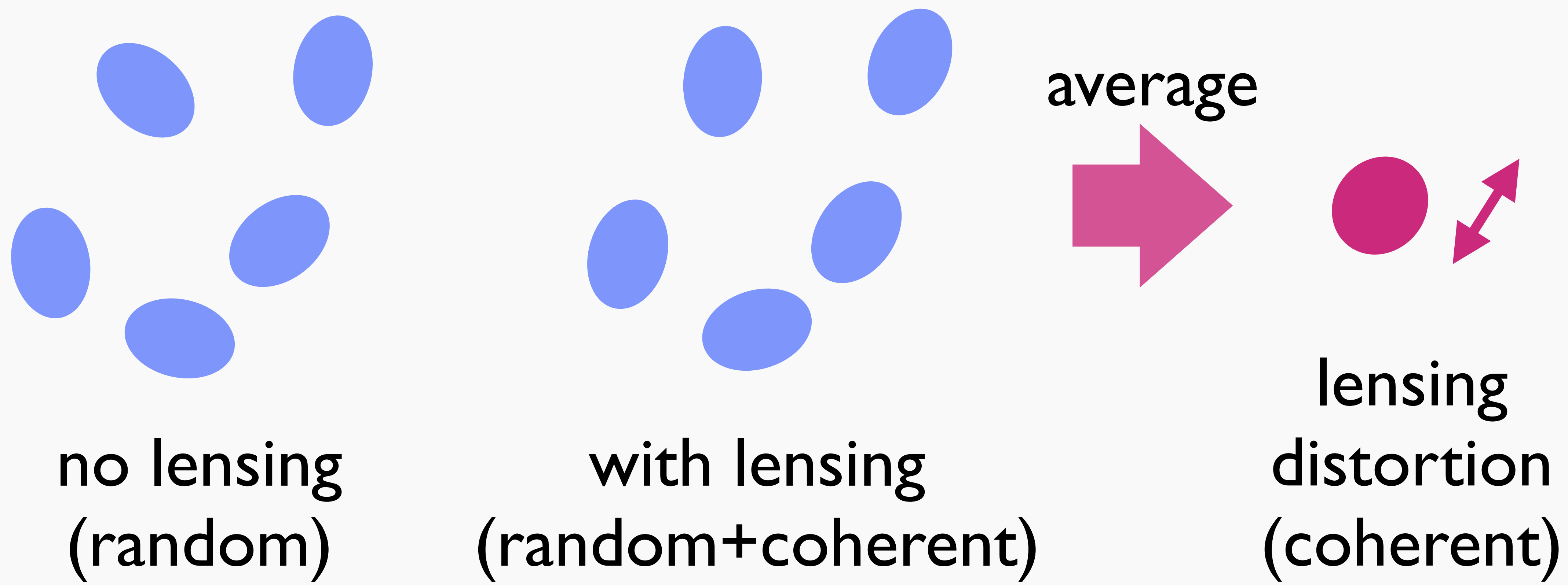


SDSS J1050+0017 (Subaru/U.Tokyo/NAOJ)

weak lensing detected only via statistical analysis

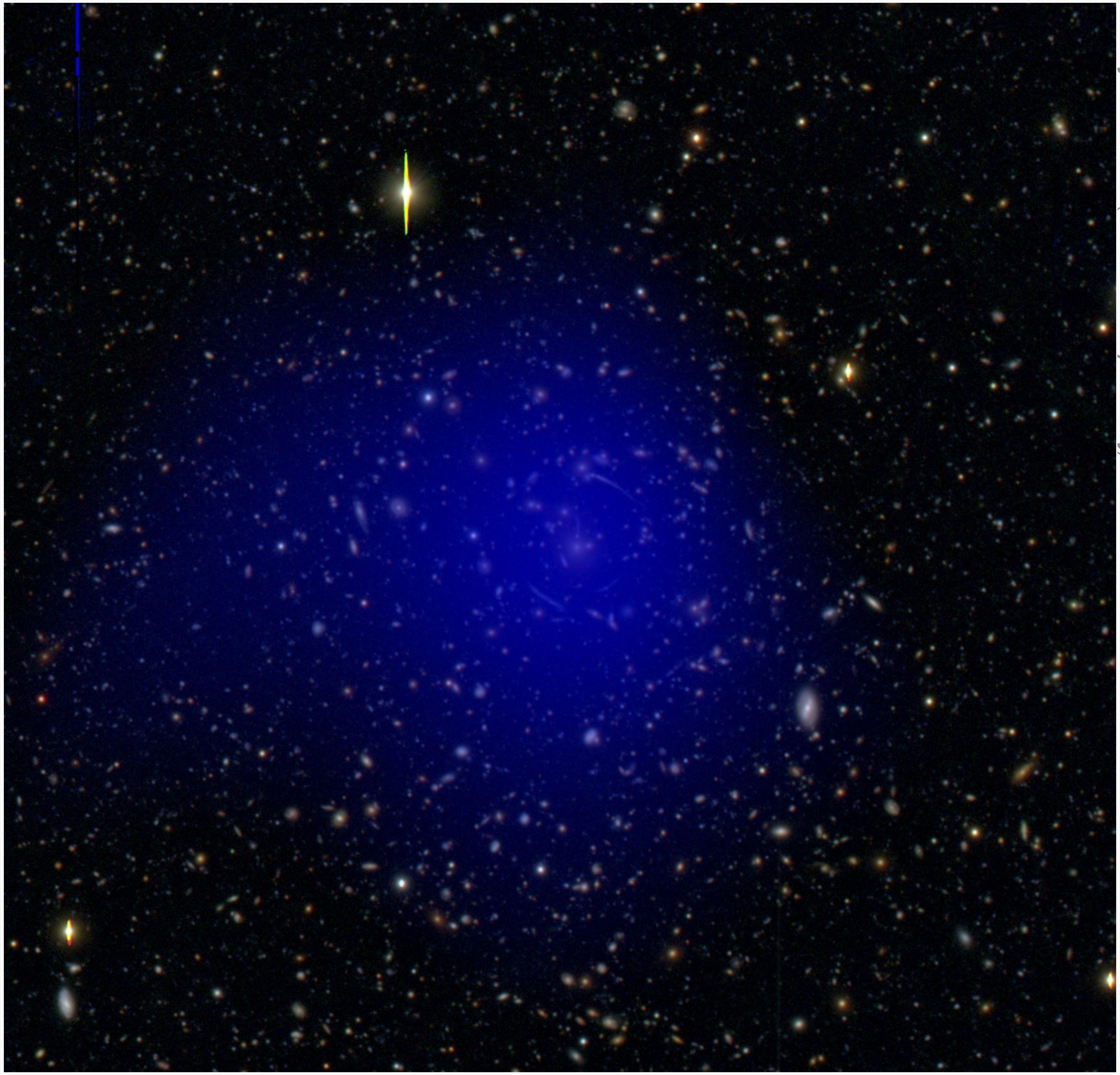
Weak (gravitational) lensing

- except for rare cases, lensing effect is **weak**
- signal is hindered by **intrinsic galaxy shapes**
- need to **average many galaxies' shapes** to extract weak gravitational lensing signals



Example of weak lensing analysis

total (dark) matter distribution inferred by weak lensing (blue)



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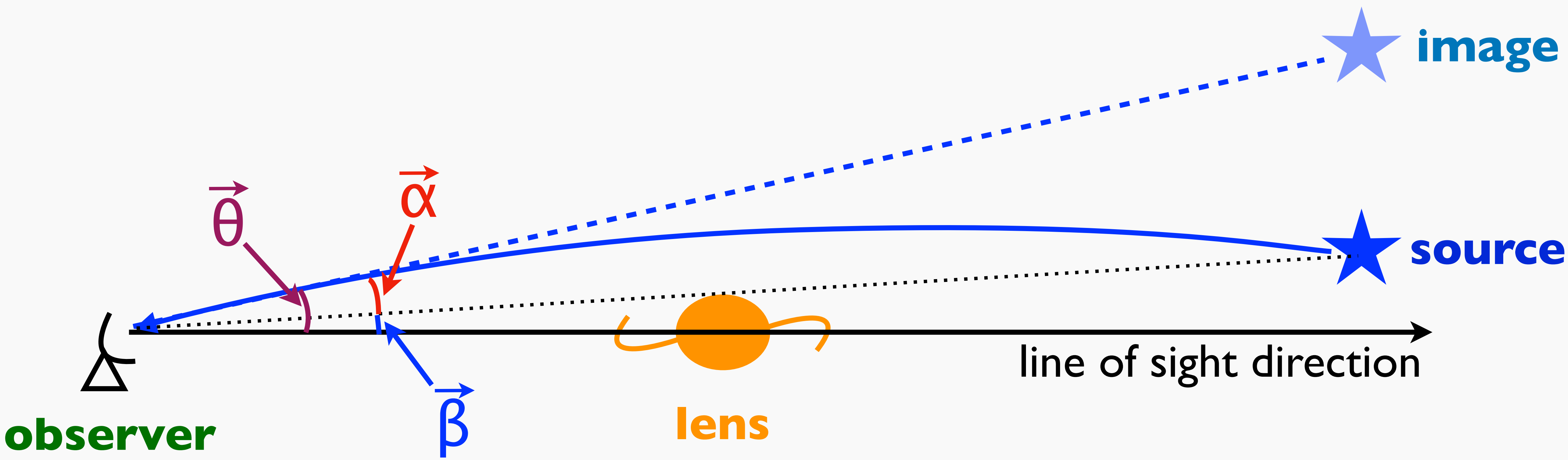
Deriving lens equation

- master equation for gravitational lensing
- derived from **geodesic equation** in general relativity (cf. Newtonian equation of motion)

$$\frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu{}_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda} = 0$$

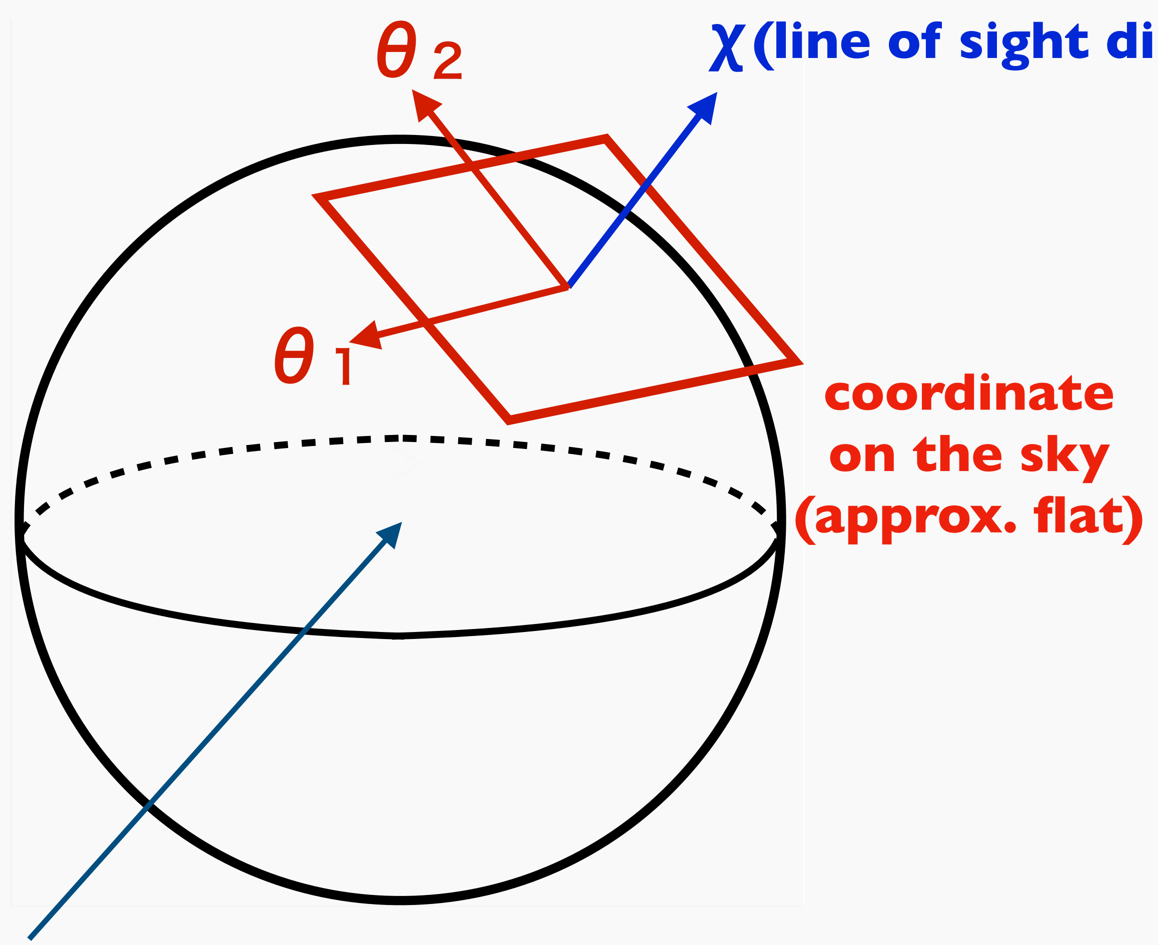
Lens equation

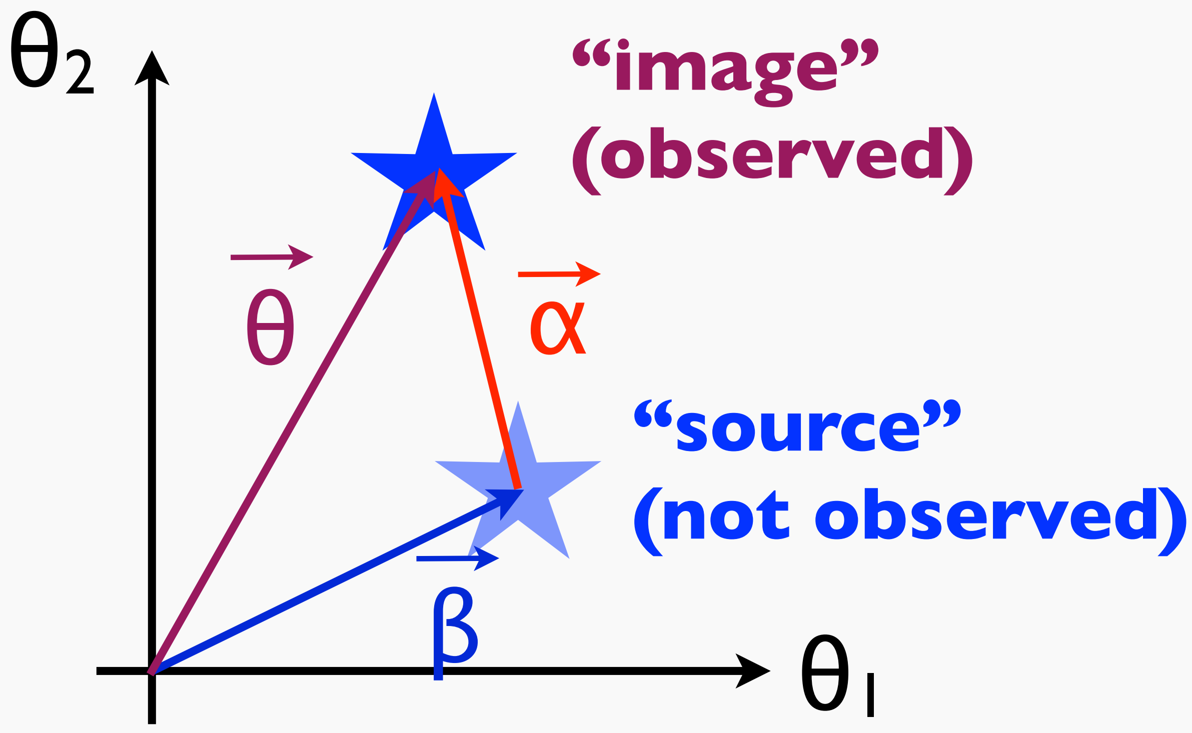


$$\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})$$

source position on the sky **image position on the sky** **deflection angle (depends on lens mass dist.)**

Lens equation





$$\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})$$

source position on the sky | image position on the sky | deflection angle (depends on lens mass dist.)

Deflection angle (thin lens approx.)

- deflection angle

$$\vec{\alpha}(\vec{\theta}) = \frac{1}{\pi}\int d\vec{\theta}'\,\kappa(\vec{\theta}')\,\frac{\vec{\theta}-\vec{\theta}'}{\left|\vec{\theta}-\vec{\theta}'\right|^2}$$

- convergence (dimensionless surface mass density of lens)

$$\kappa(\vec{\theta}) = \frac{\Sigma(\vec{\theta})}{\Sigma_{\rm crit}}$$

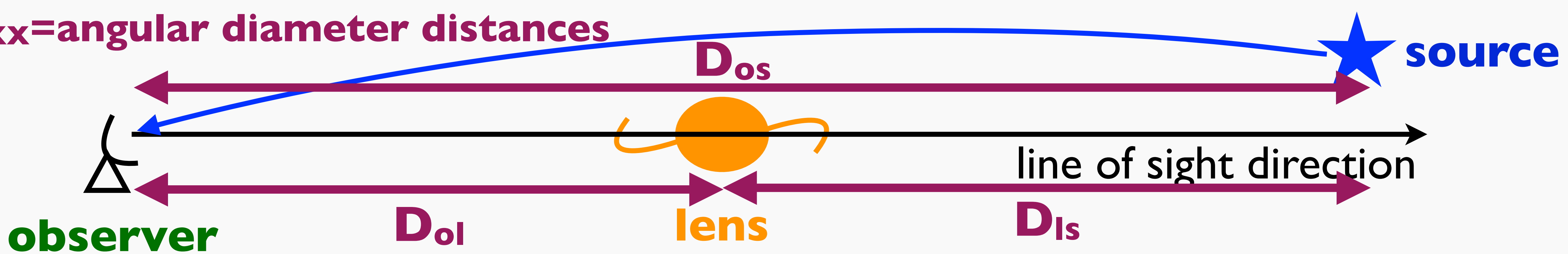
$$\Sigma(\vec{\theta}) = \int dz\,\rho(D_{\rm ol}\vec{\theta}, z)$$

density profile ρ projected along line of sight

$$\Sigma_{\rm crit} = \frac{c^2}{4\pi G}\frac{D_{\rm os}}{D_{\rm ol}D_{\rm ls}}$$

critical surface mass density

Dxx=angular diameter distances

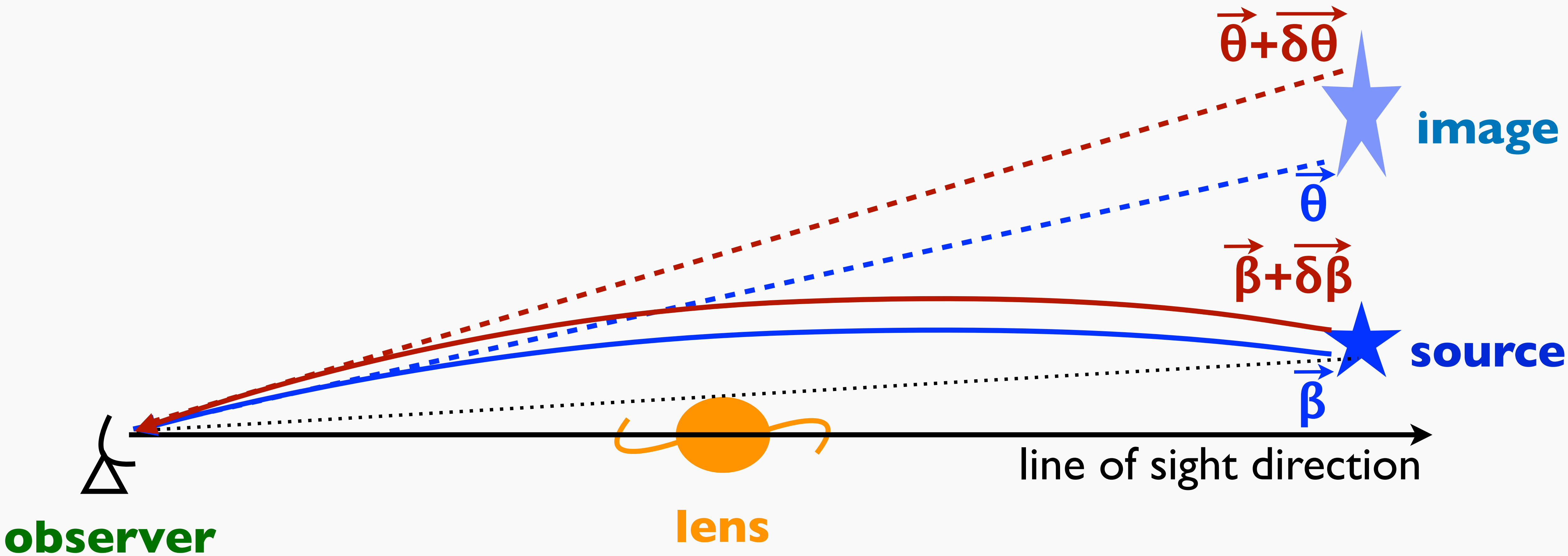


Lens equation: summary

$$\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})$$

- describes mapping between **source position $\vec{\beta}$** (not observed) and **image position $\vec{\theta}$** (observed)
- **deflection angle $\vec{\alpha}$** is determined by the mass distribution of the lens = **convergence κ**

Lensing of an extended source

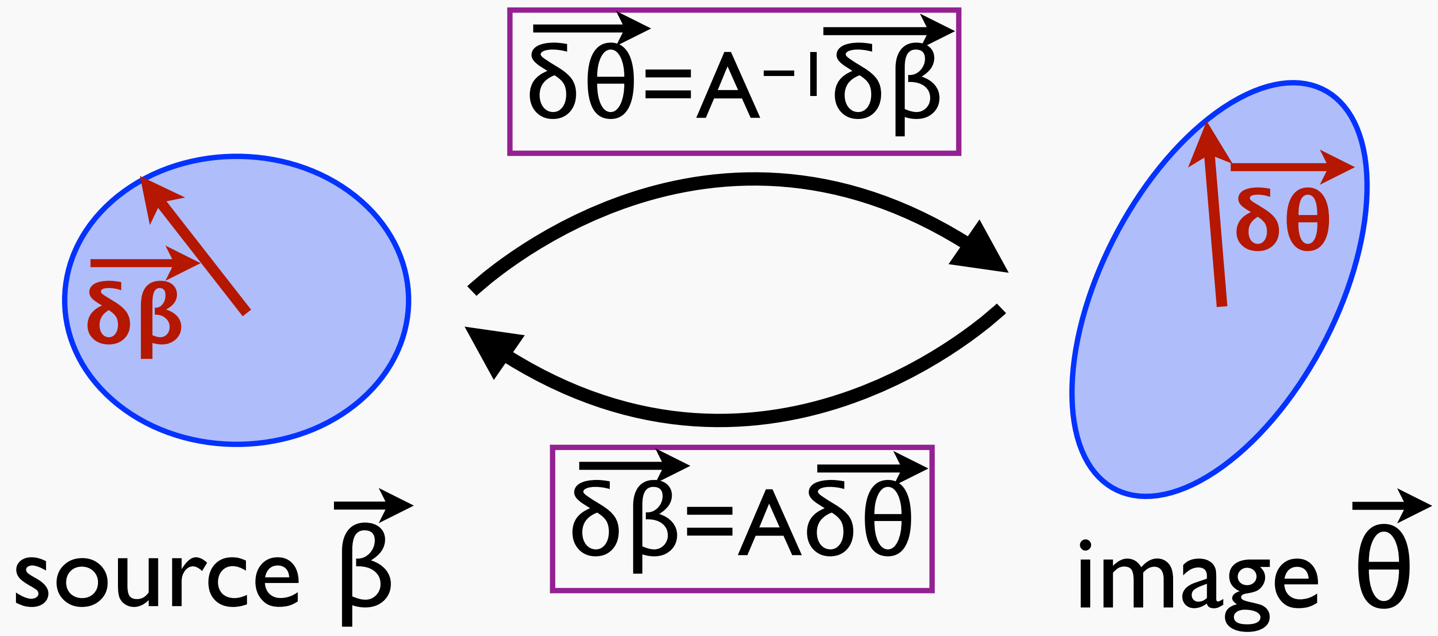


$$\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})$$

$$\begin{aligned}\vec{\beta} + \overrightarrow{\delta\beta} &= \vec{\theta} + \overrightarrow{\delta\theta} - \vec{\alpha}(\vec{\theta} + \overrightarrow{\delta\theta}) \\ &\simeq \vec{\theta} + \overrightarrow{\delta\theta} - \vec{\alpha}(\vec{\theta}) - \frac{\partial \vec{\alpha}}{\partial \vec{\theta}} \overrightarrow{\delta\theta}\end{aligned}$$

Distortion of shape

$$\overrightarrow{\delta\beta} = A\,\overrightarrow{\delta\theta} \qquad A = I - \frac{\partial\vec{\alpha}}{\partial\vec{\theta}} = \begin{pmatrix} 1 - \frac{\partial\alpha_1}{\partial\theta_1} & -\frac{\partial\alpha_1}{\partial\theta_2} \\ -\frac{\partial\alpha_2}{\partial\theta_1} & 1 - \frac{\partial\alpha_2}{\partial\theta_2} \end{pmatrix}$$



A : *de*-lensing
A^{-1}: lensing

Connection with convergence

- using the relation

$$\frac{\partial}{\partial \vec{\theta}} \left(\frac{\vec{\theta} - \vec{\theta}'}{\left| \vec{\theta} - \vec{\theta}' \right|^2} \right) = 2\pi \delta^{\mathrm{D}}(\vec{\theta} - \vec{\theta}')$$

Dirac delta function

we can show

$$\mathrm{tr}(A) = 2 - \frac{\partial \alpha_1}{\partial \theta_1} - \frac{\partial \alpha_2}{\partial \theta_2} = 2 - 2\kappa(\vec{\theta})$$

convergence

(dimensionless surface mass density of lens)

Convergence and shear

$$\overrightarrow{\delta\beta} = A\,\overrightarrow{\delta\theta} \qquad A = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$

convergence

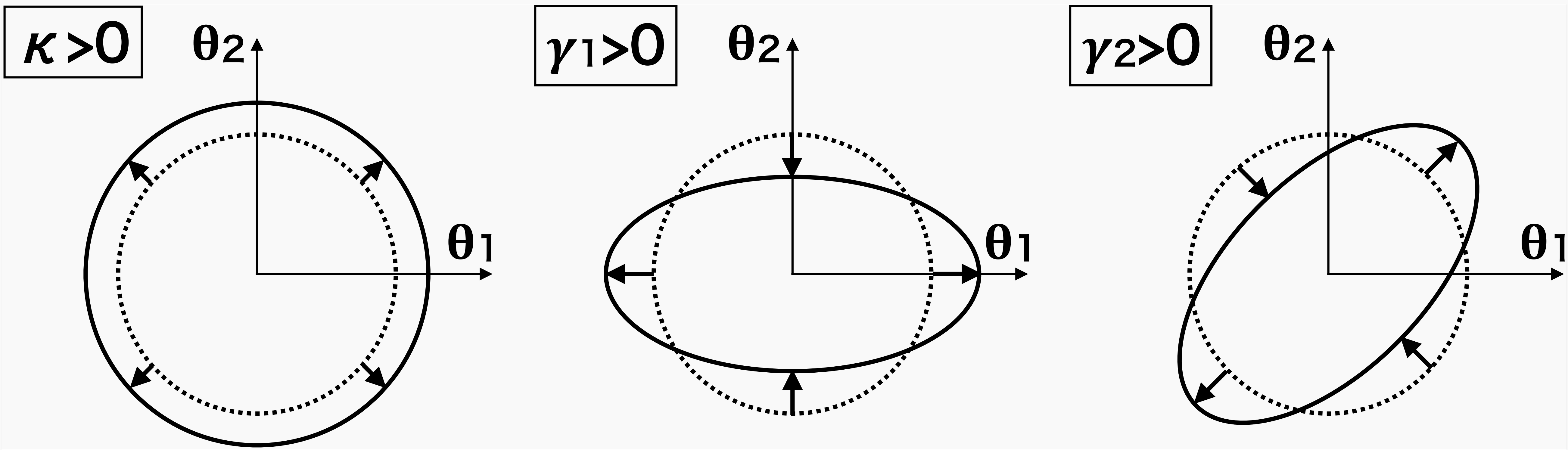
$$\kappa(\vec{\theta}) = \frac{\Sigma(\vec{\theta})}{\Sigma_{\rm crit}}$$

shear

$$\gamma_1(\vec{\theta}) = \frac{1}{2}\left(\frac{\partial\alpha_1}{\partial\theta_1} - \frac{\partial\alpha_2}{\partial\theta_2}\right) = \frac{1}{\pi}\int d\vec{\theta}'\,\kappa(\vec{\theta}')\,\frac{(\theta_2-\theta_2')^2-(\theta_1-\theta_1')^2}{\left\{(\theta_1-\theta_1')^2+(\theta_2-\theta_2')^2\right\}^2}$$

$$\gamma_2(\vec{\theta}) = \frac{\partial\alpha_1}{\partial\theta_2} = \frac{\partial\alpha_2}{\partial\theta_1} = \frac{1}{\pi}\int d\vec{\theta}'\,\kappa(\vec{\theta}')\,\frac{-2(\theta_1-\theta_1')(\theta_2-\theta_2')}{\left\{(\theta_1-\theta_1')^2+(\theta_2-\theta_2')^2\right\}^2}$$

Weak lensing distortions



convergence κ

difficult to measure

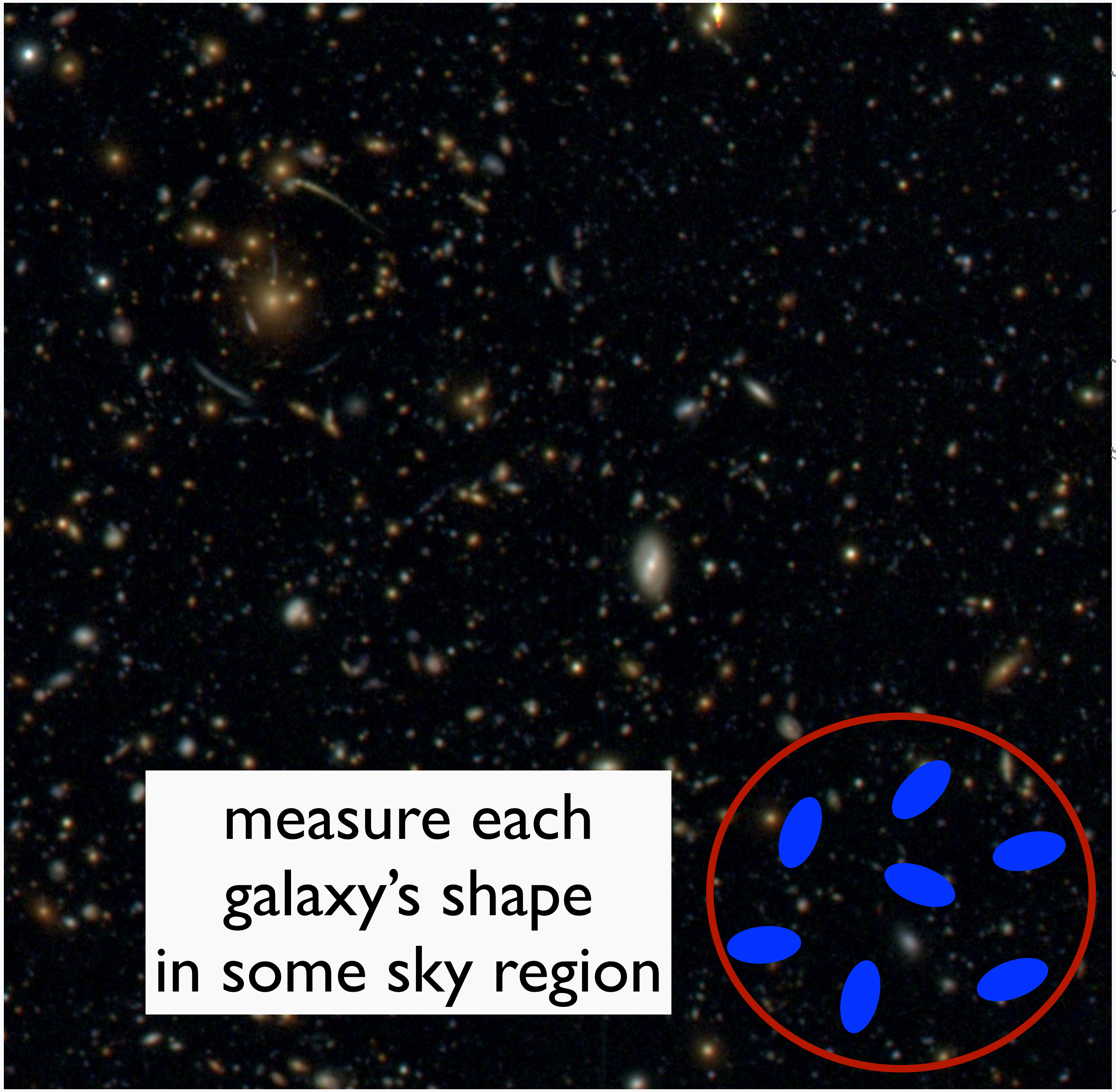
shear γ

can be measured by statistical analysis of galaxy shapes

Measuring shear

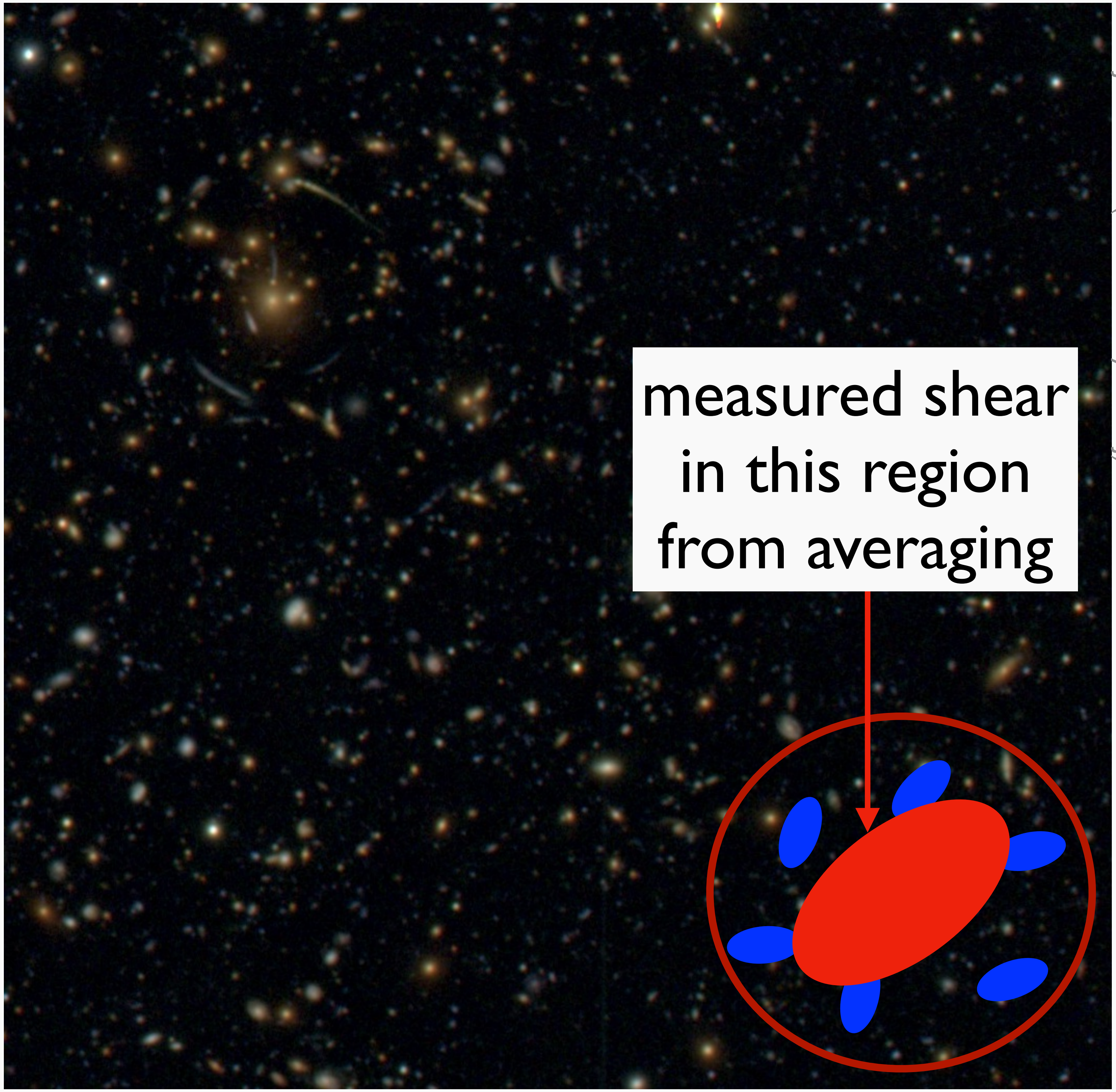
- each (j-th) galaxy have intrinsic shape ϵ_i^j (i=1, 2)
- observed shape is affected by weak lensing distortion $\epsilon_i^{\mathrm{obs},j} = \epsilon_i^j + \gamma_i$
- *assume* that orientations of galaxies are random on average $\langle \epsilon_i^j \rangle \approx \frac{1}{N} \sum_j \epsilon_i^j = 0$
- shear is measured by **averaging observed galaxy shapes** $\langle \epsilon_i^{\mathrm{obs},j} \rangle \approx \frac{1}{N} \sum_j \epsilon_i^{\mathrm{obs},j} = \gamma_i$

Measuring shear



SDSS J1050+0017 (Subaru/U.Tokyo/NAOJ)

Measuring shear



SDSS J1050+0017 (Subaru/U.Tokyo/NAOJ)

Shear is small

- weak lensing shear is typically very small

$$\epsilon_i^{\mathrm{obs},j} = \epsilon_i^j + \gamma_i$$

intrinsic galaxy shape ≈ 0.3

weak lensing shear typically ≈ 0.03–0.003

- measurement noise from intrinsic galaxy shapes

$$\frac{\mathrm{S}}{\mathrm{N}} = \frac{\gamma_i}{\sqrt{\langle \epsilon_i^2 \rangle / \sqrt{N}}}$$

need N ≳ 10^{3-4} galaxies for significant detection

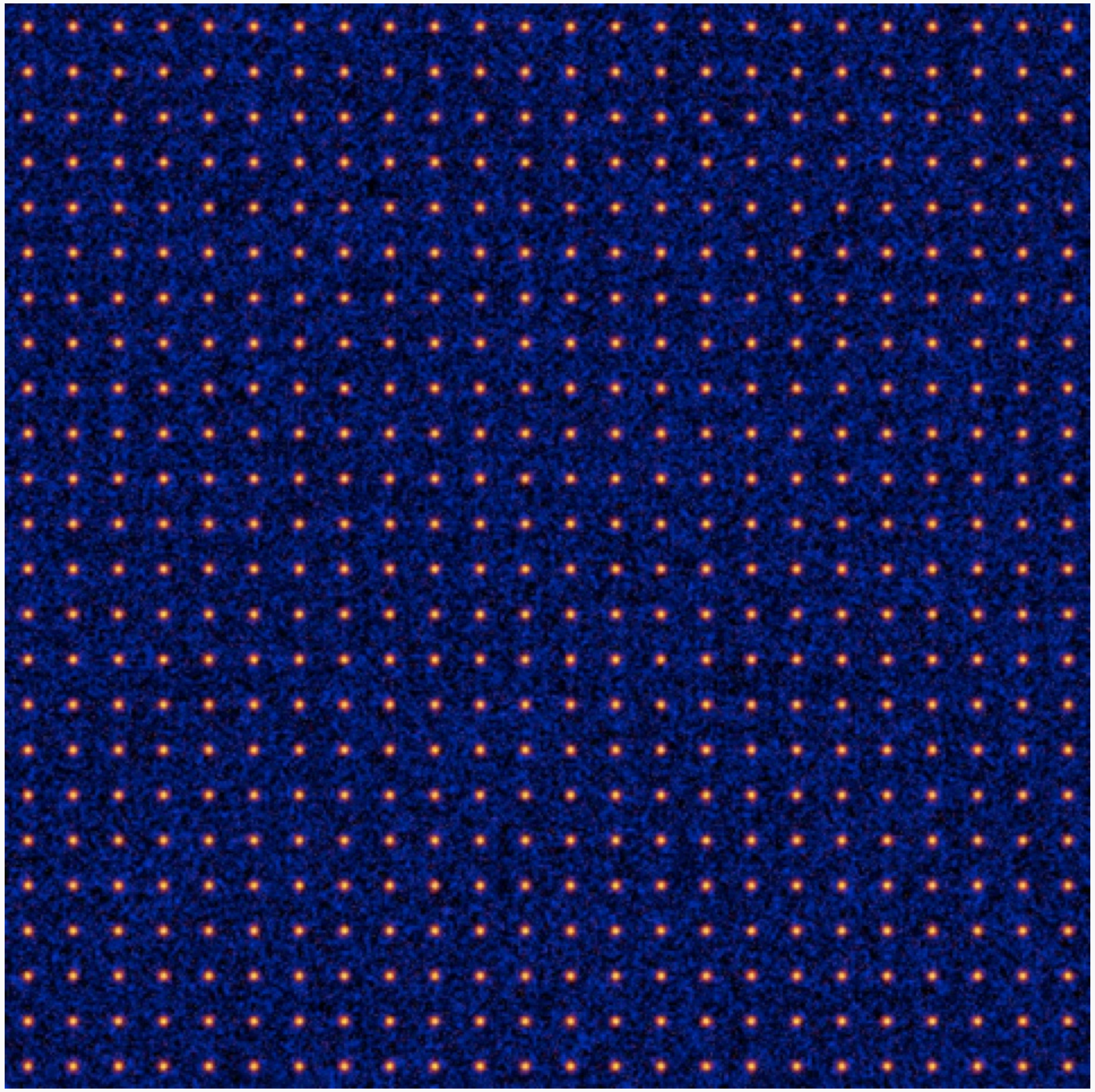
number of galaxies averaged

Convergence and shear: summary

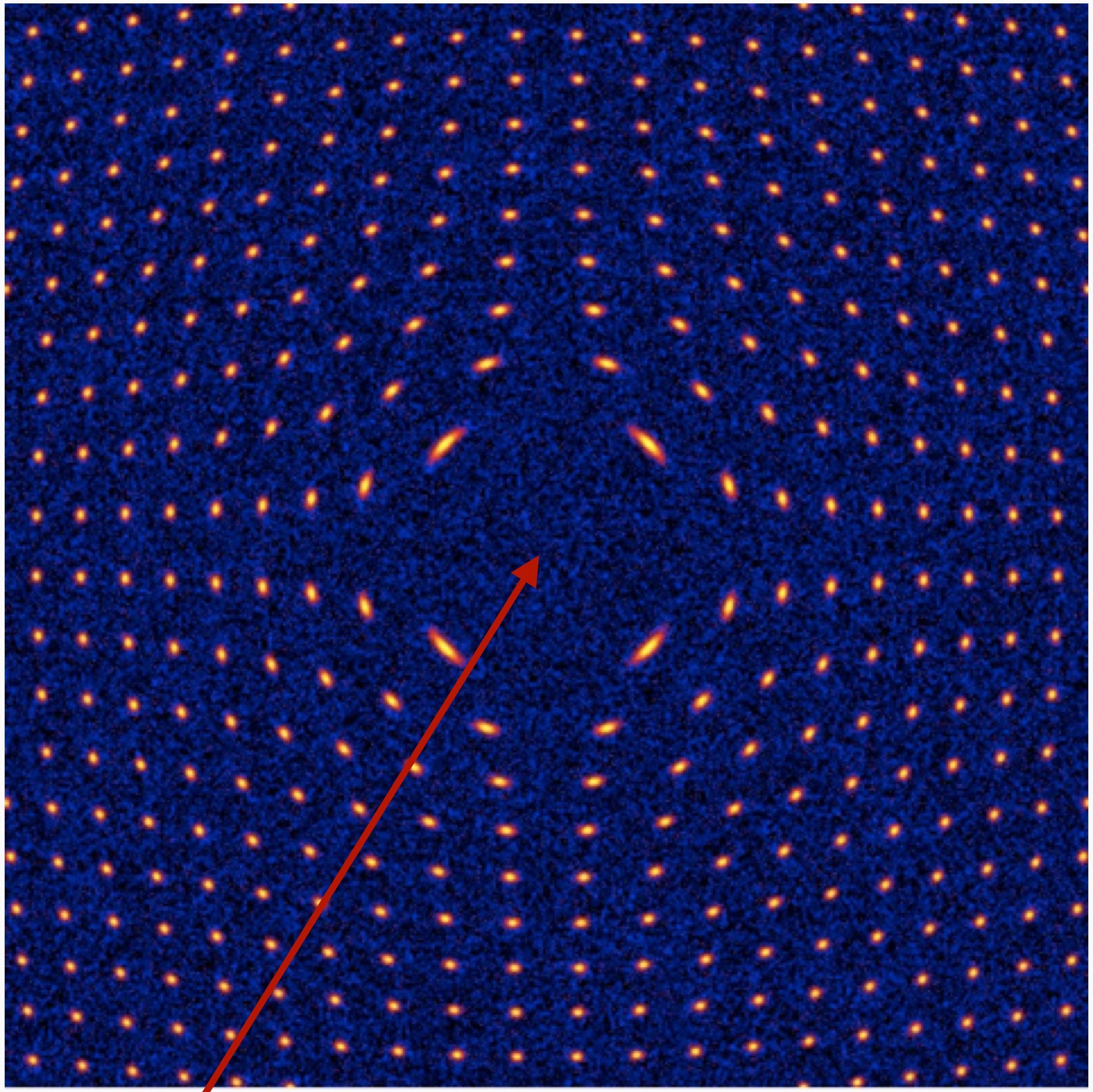
- galaxy shapes are affected by weak lensing
- convergence induces uniform expansion, shear induces distortions
- shear can be calculated from convergence
- shear is measured in observations by averaging many galaxies' shapes

simulated by *glafic*

Simulation of lensing distortion



without lensing

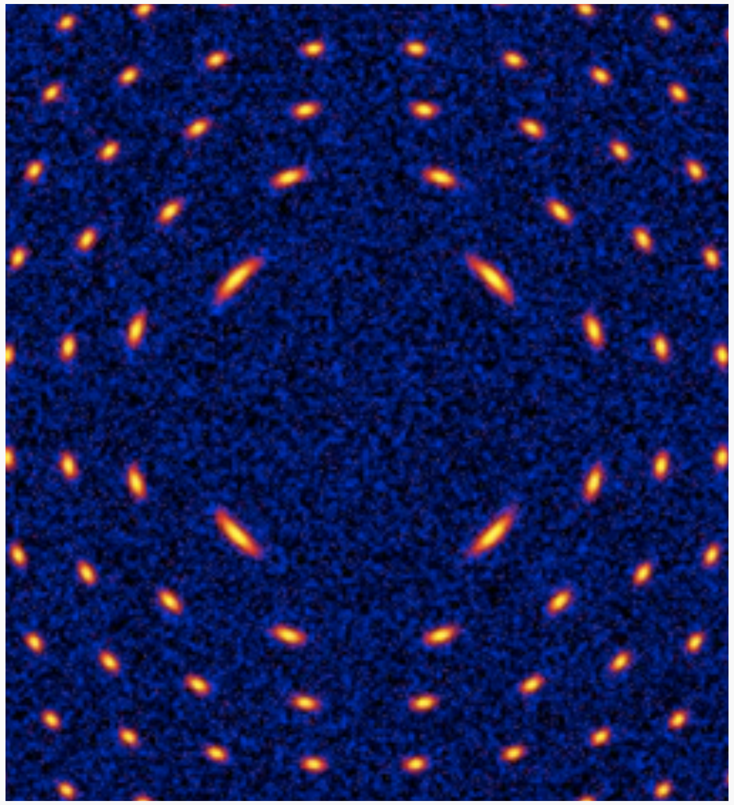


with lensing

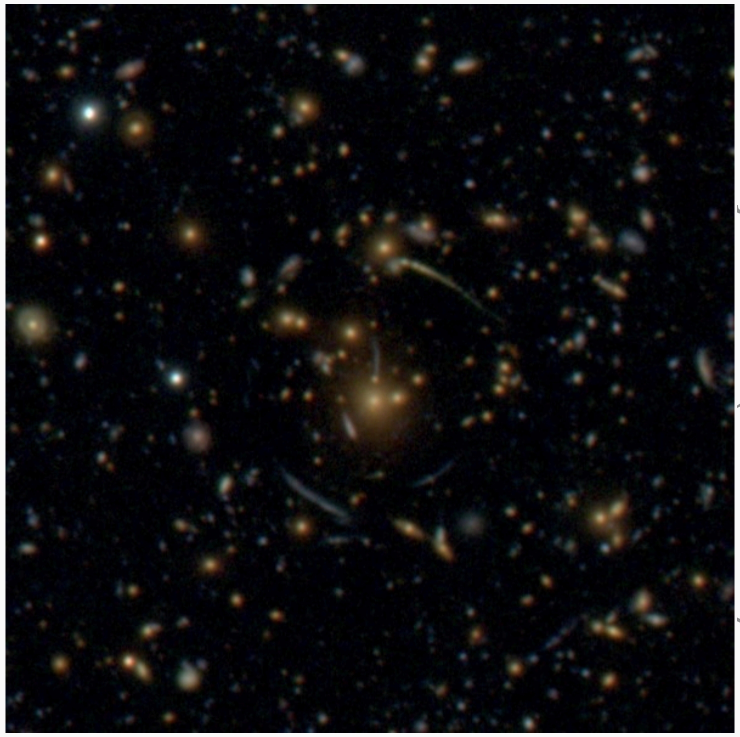
lens center here

Tangential shear

- high density lens distorts shapes of background galaxies along **tangential** direction
- true both for strong and weak lensing
- measure lens mass dist. from **tangential shear**



simulation



observation

SDSS J1050+0017 (Subaru/U.Tokyo)

Calculation of tangential shear

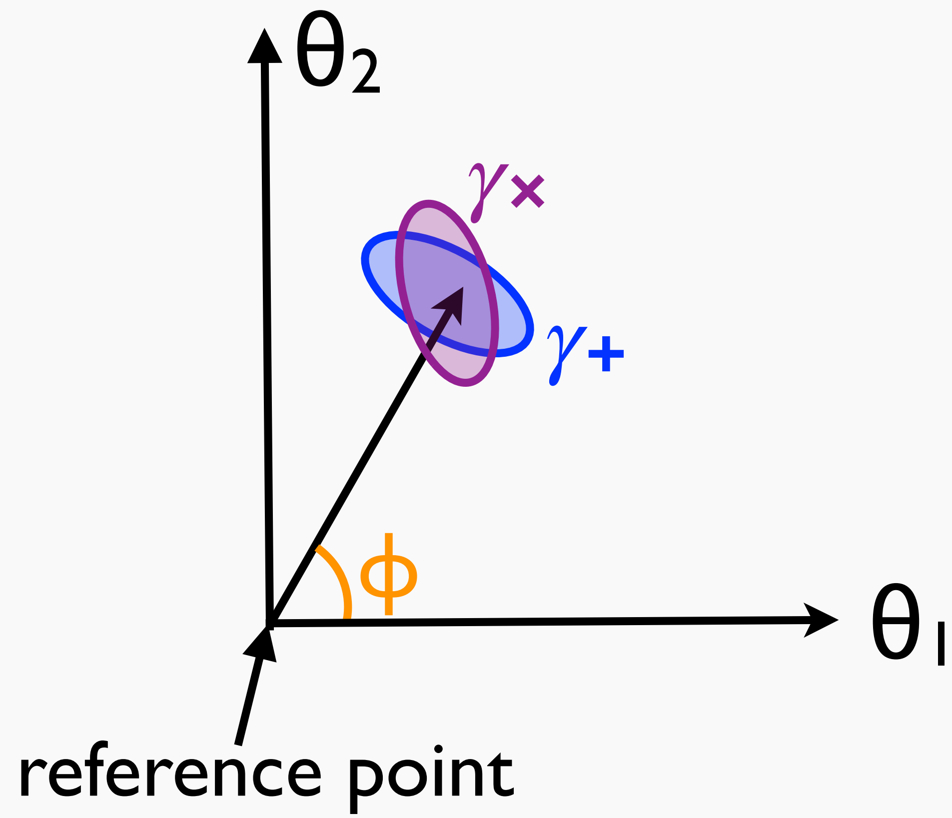
- for a given reference point, tangential and cross shear is defined by

tangential shear

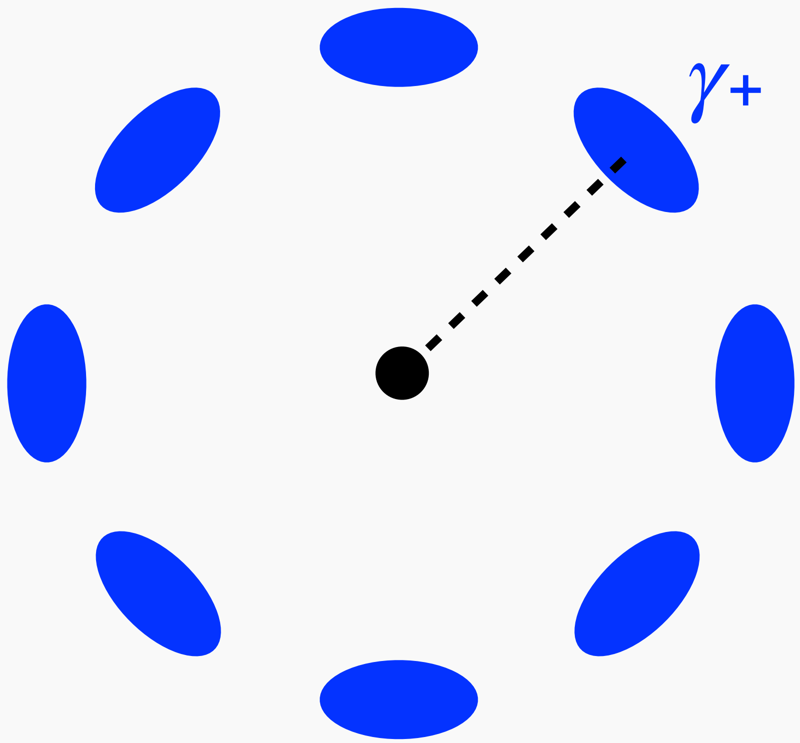
$$\gamma_+ = -\gamma_1 \cos 2\phi - \gamma_2 \sin 2\phi$$

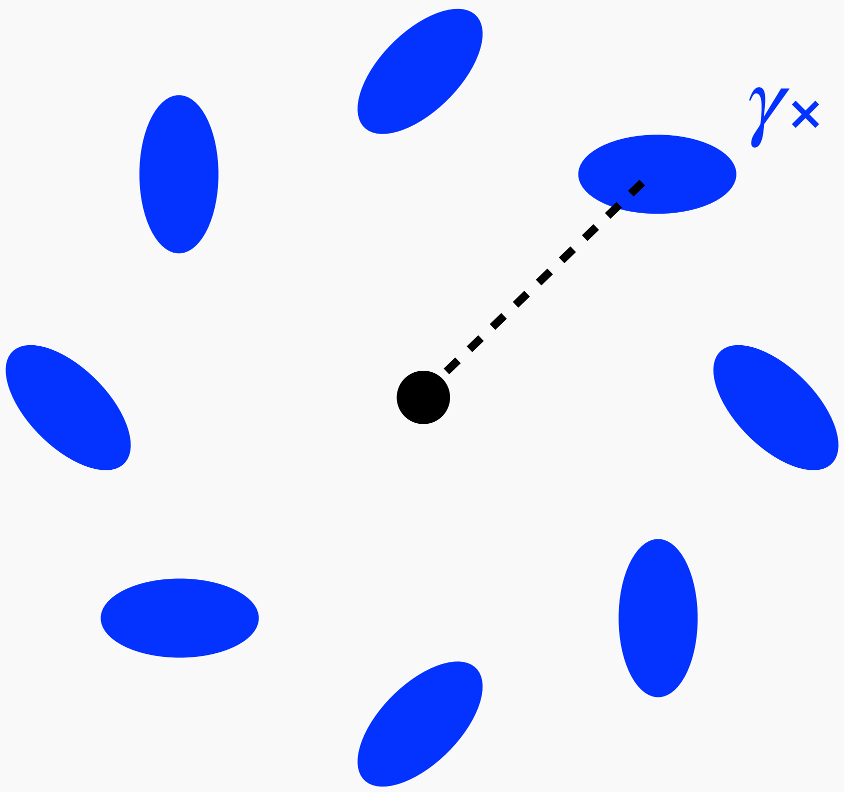
cross shear (45 degree rotated)

$$\gamma_\times = \gamma_1 \sin 2\phi - \gamma_2 \cos 2\phi$$



Tangential and cross shear





tangential shear
generated by lensing

cross shear
not generated by lensing,
used for checking systematics

Calculations

- from the definition of γ_1 and γ_2, it is shown

(circular symmetric κ, reference point = κ center)

tangential shear

$$\gamma_+(\theta) = \bar{\kappa}(< \theta) - \kappa(\theta) = \frac{2}{\theta^2}\int_0^\theta d\theta'\,\theta'\,\kappa(\theta') - \kappa(\theta)$$

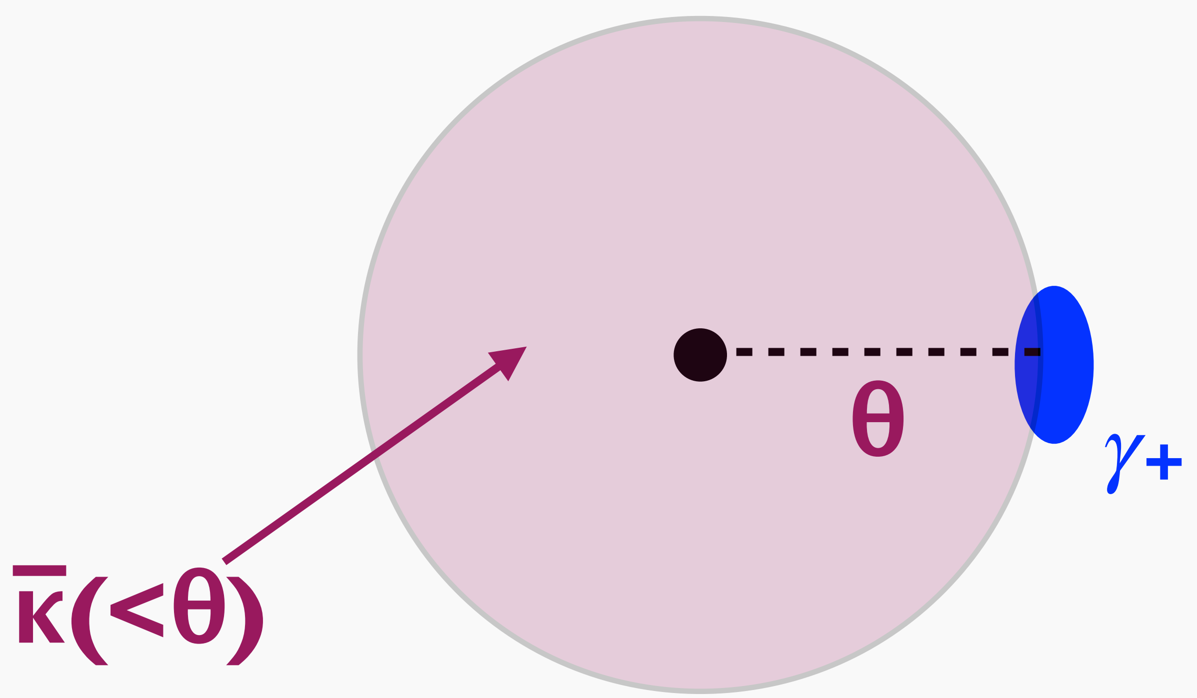
cross shear

$$\gamma_\times(\theta) = 0$$

Note: shear is non-local

$$\gamma_+(\theta) = \bar{\kappa}(< \theta) - \kappa(\theta) = \frac{2}{\theta^2}\int_0^\theta d\theta'\,\theta'\,\kappa(\theta') - \kappa(\theta)$$

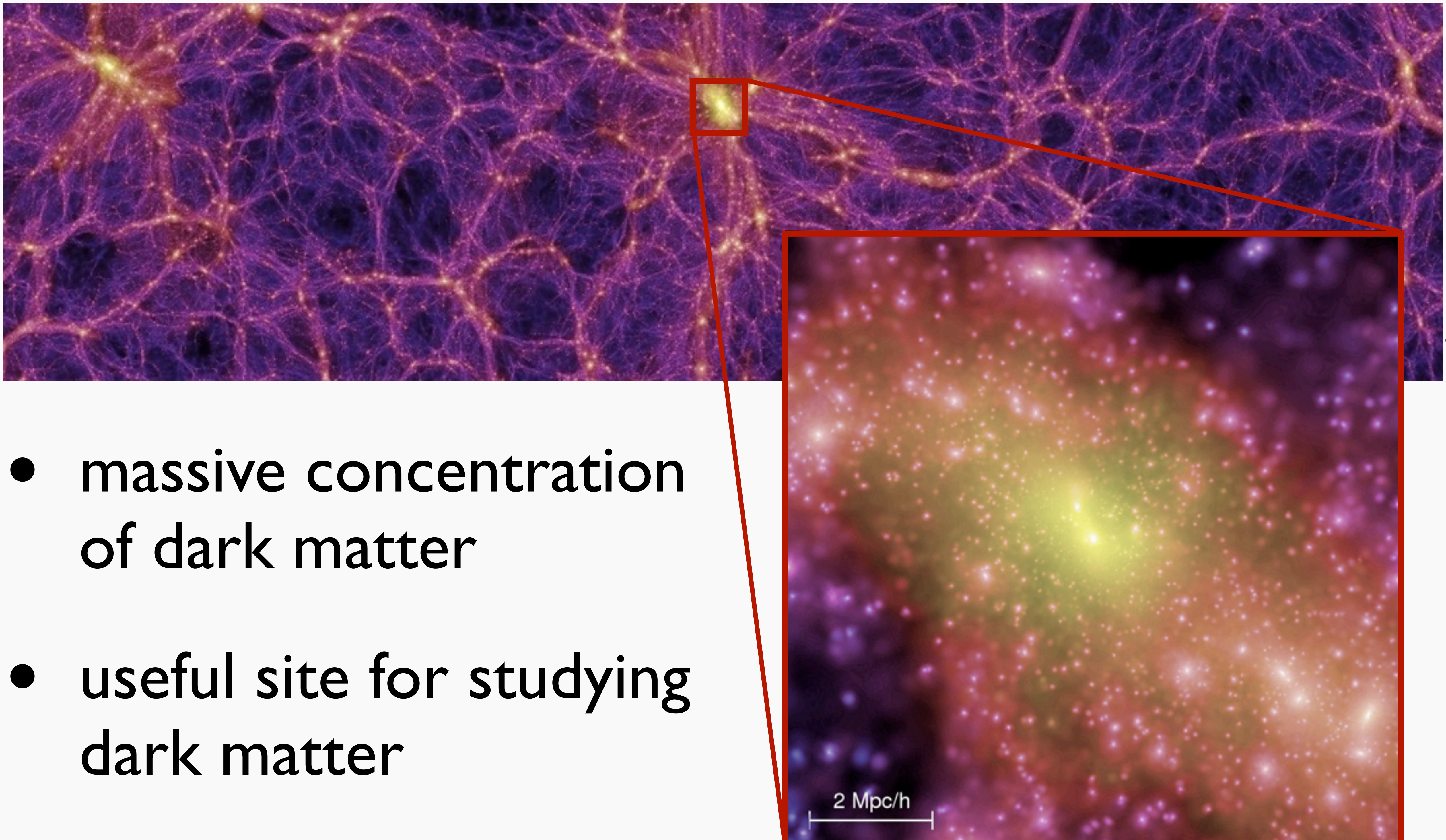
- tangential shear at θ is determined by integrated mass at < θ, not just by mass density at θ



Tangential shear: summary

- gravitational lensing induces coherent tangential distortions around the lens
- tangential shear at some radius depends on integrated mass within that radius
- cross (45 degree rotated) shear vanishes and thus used to check systematic errors

Galaxy cluster

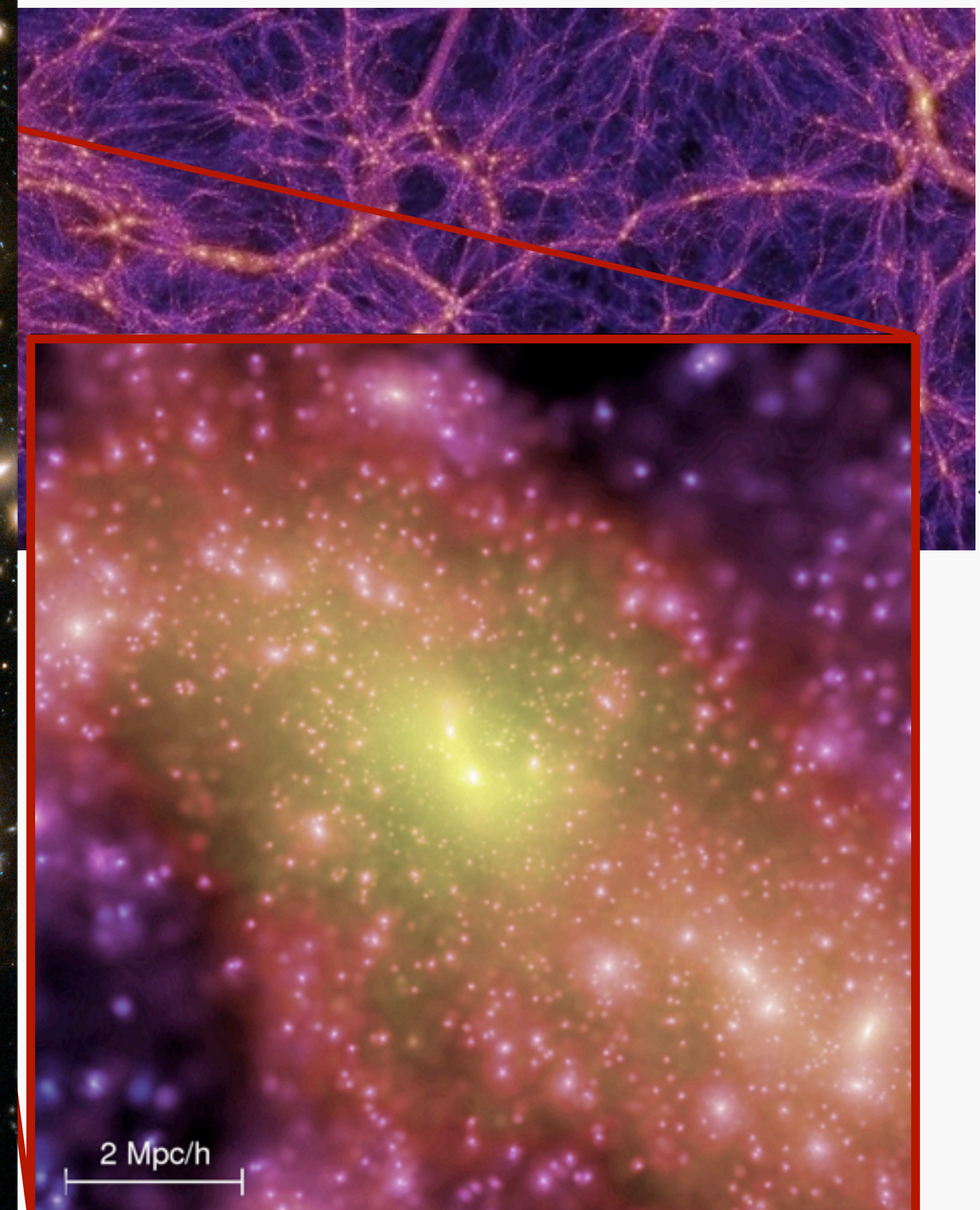


Millennium Simulation Project

- massive concentration of dark matter
- useful site for studying dark matter

Galaxy cluster

Abell 370, NASA/STScI

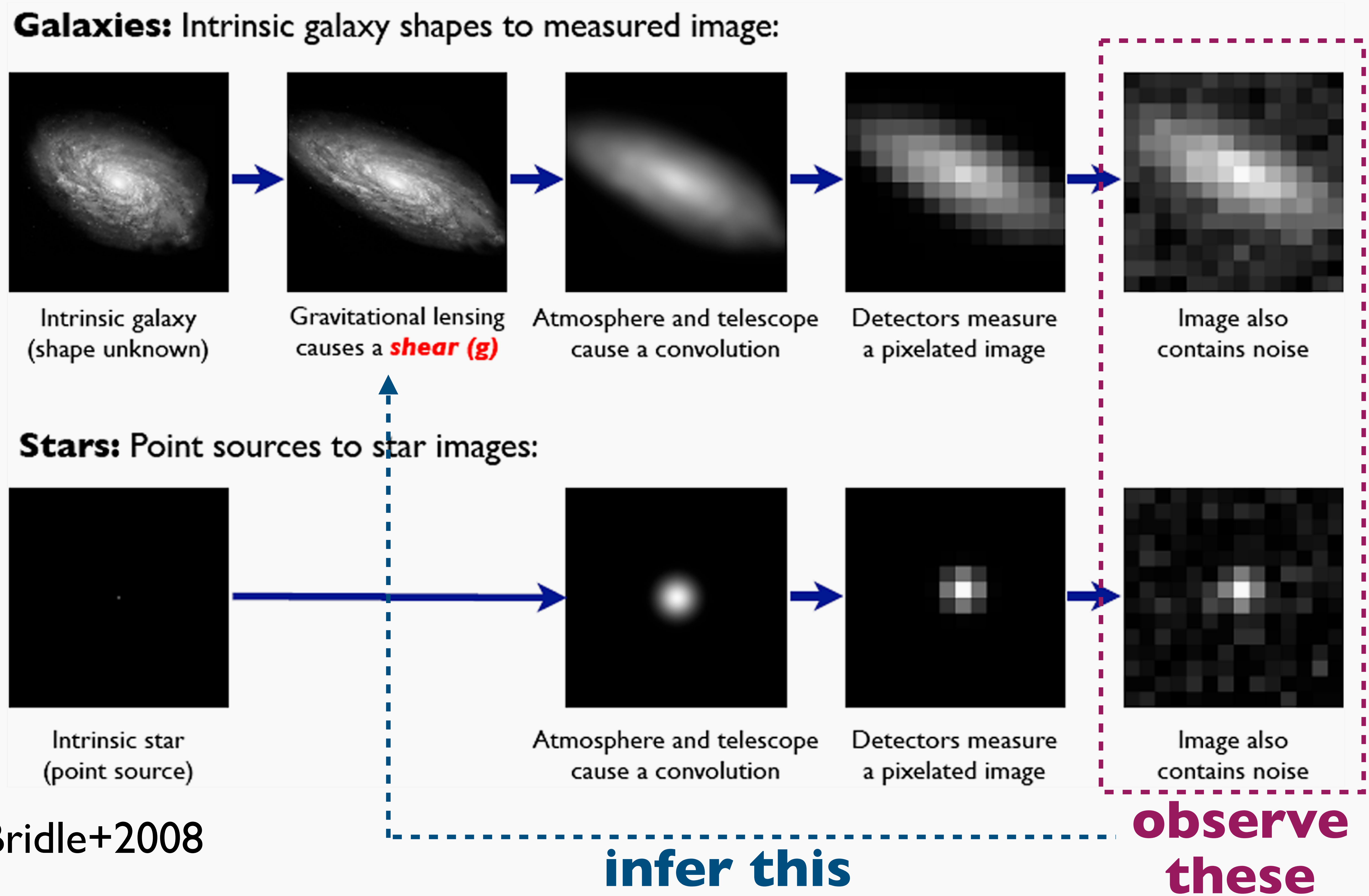


Millennium Simulation Project

Cluster weak lensing analysis

- cluster is dark matter dominated system, which has been extensively studied using weak lensing
- I show an example of cluster weak lensing analysis based on tangential shear

Shape measurement



Bridle+2008

Measuring tangential shear

- define an annulus around radius θ

- average tangential shear of all galaxies in the annulus

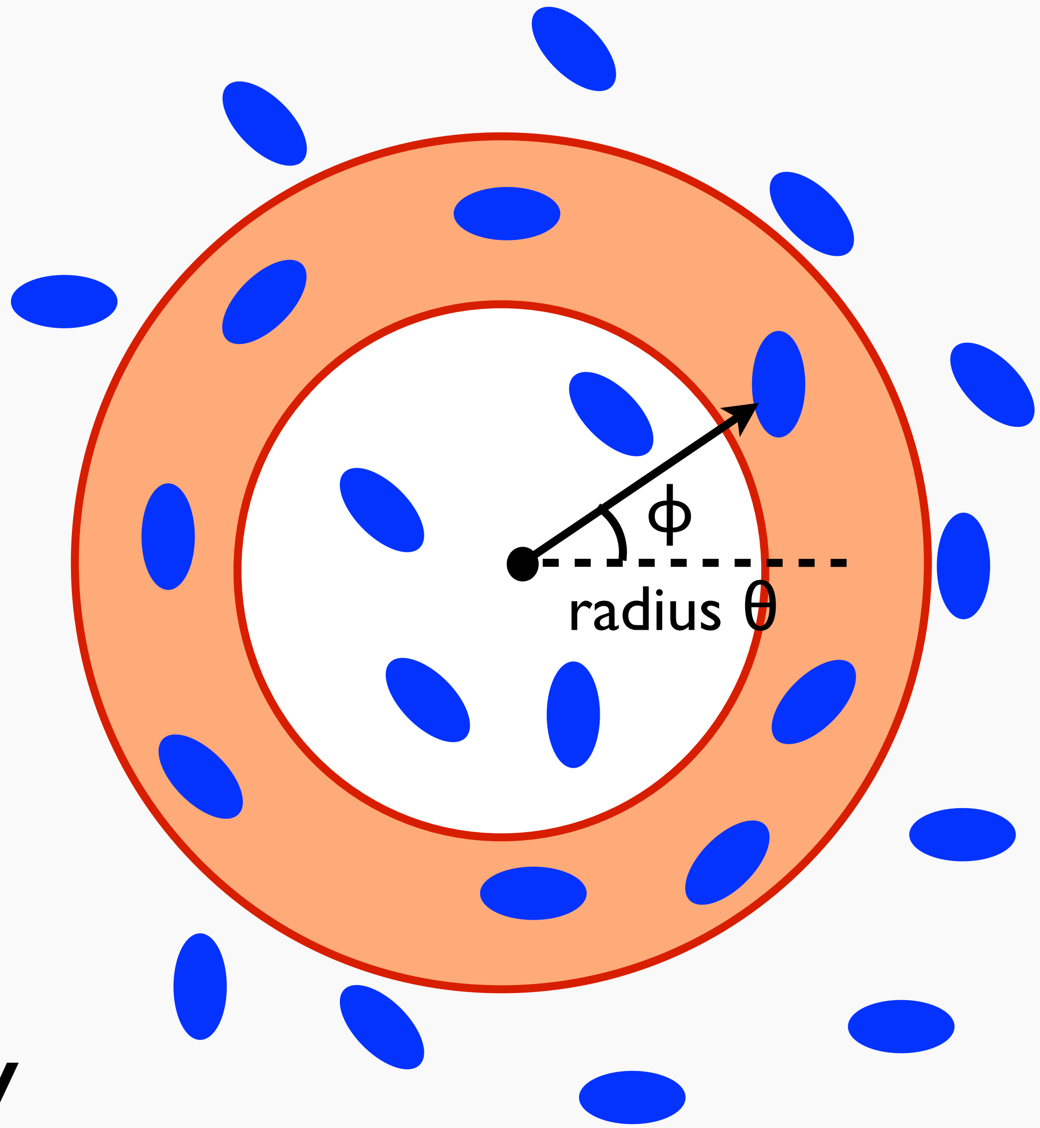
$$\bar{\gamma}_+(\theta) = \frac{\sum_j w_j \gamma_{+,j}}{\sum_j w_j}$$

j: label of galaxies in the bin

w_j: weight of j-th galaxy

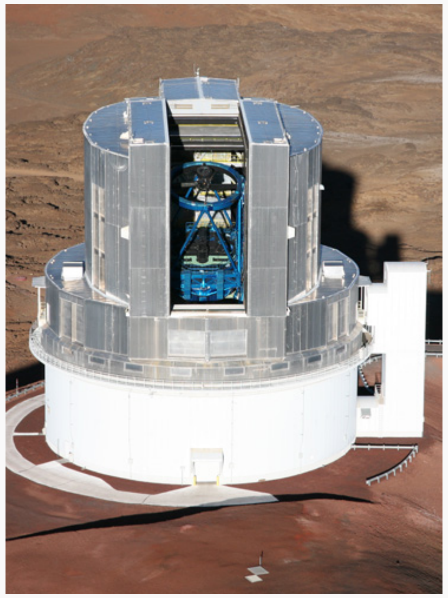
- its error is approx. given by

$$\sigma \approx \sqrt{\frac{\sum_j w_j(\gamma_{+,j} - \bar{\gamma}_+)^2}{\sum_j w_j} \frac{\sum_j w_j^2}{\left(\sum_j w_j\right)^2}}$$



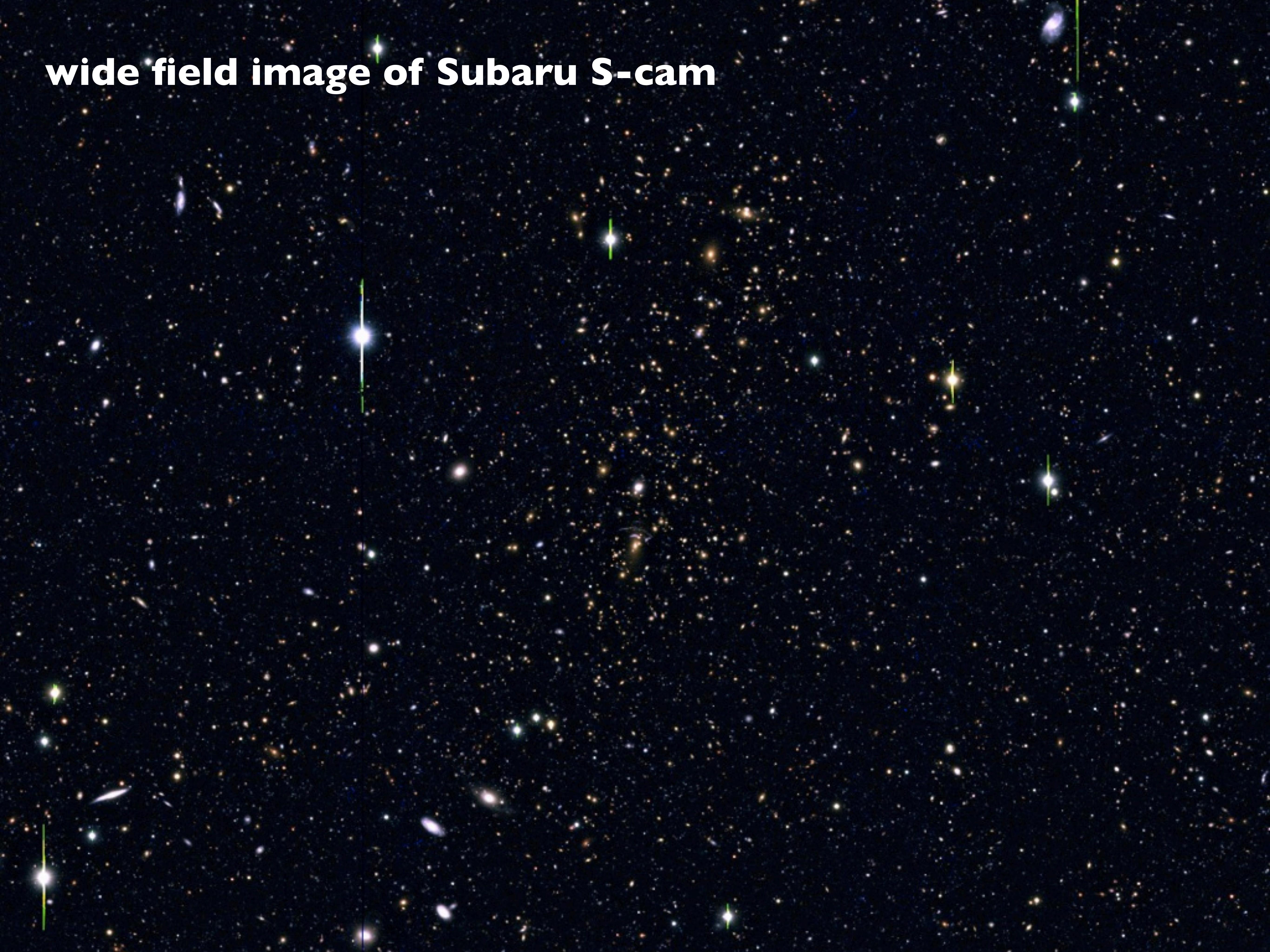
Example: SDSSJ1138+2754

- massive cluster at z=0.45
- observed with Subaru Suprime-cam (MO+2012)

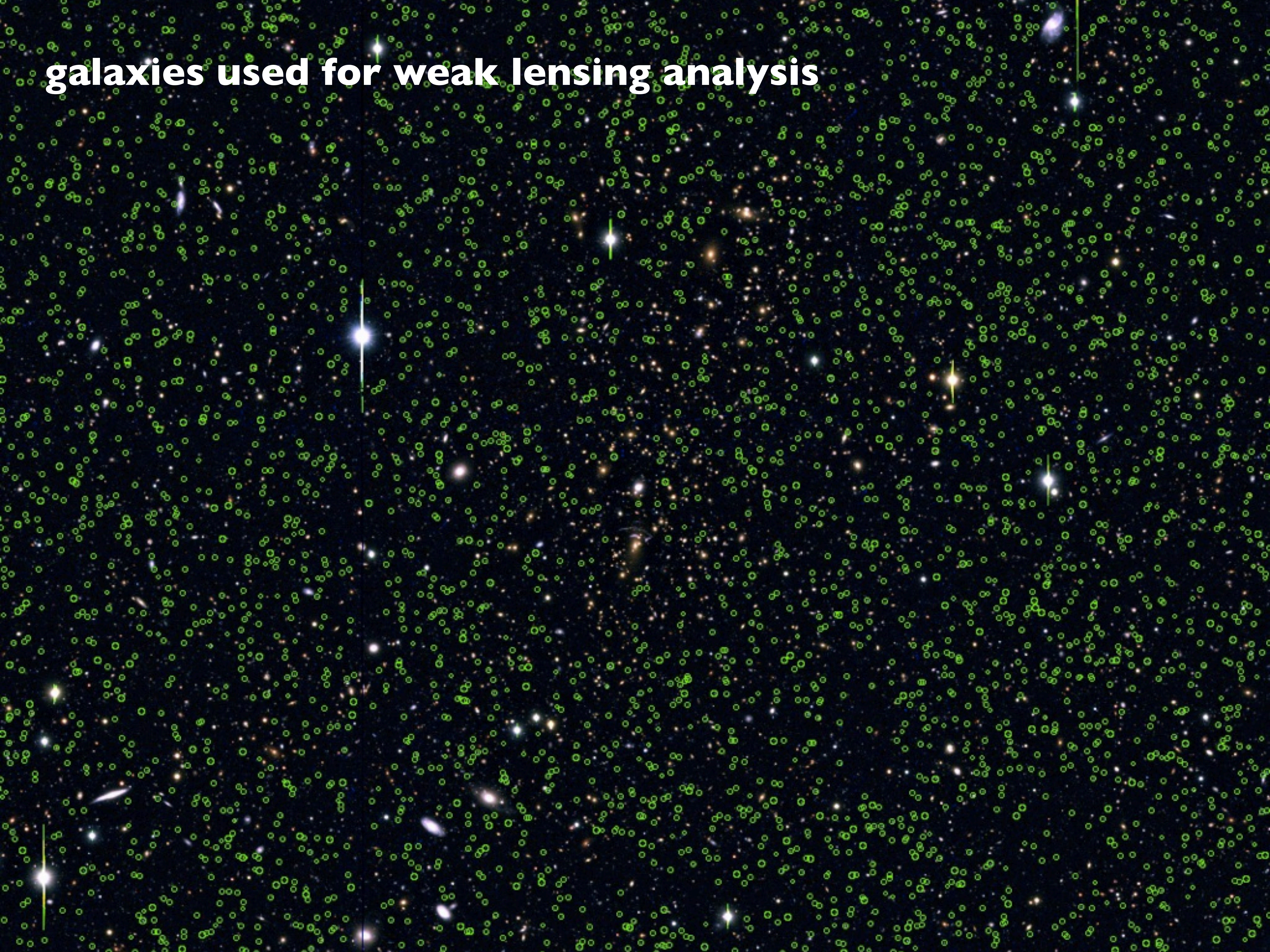


Subaru/Suprime-cam gri-band

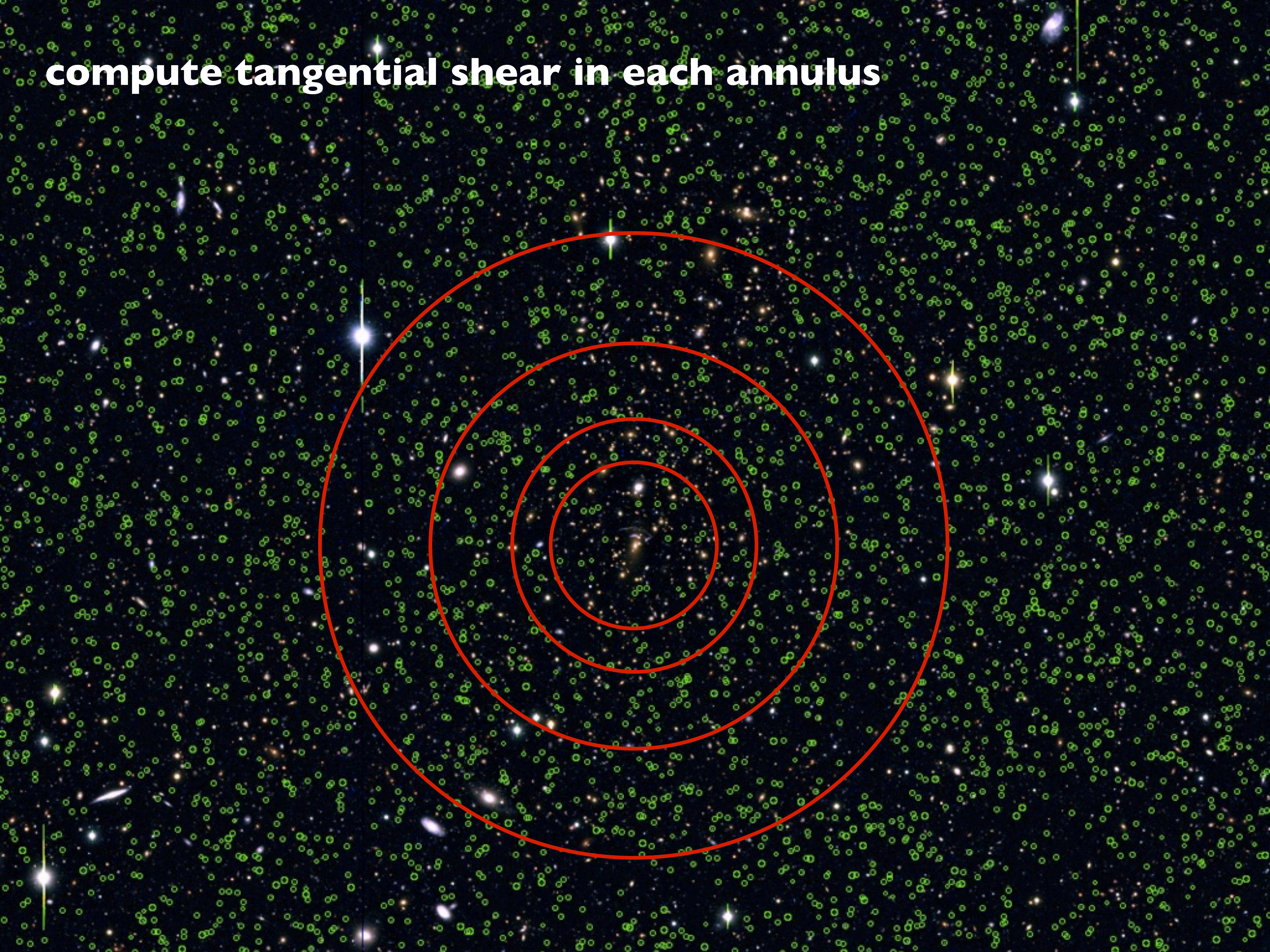
wide field image of Subaru S-cam



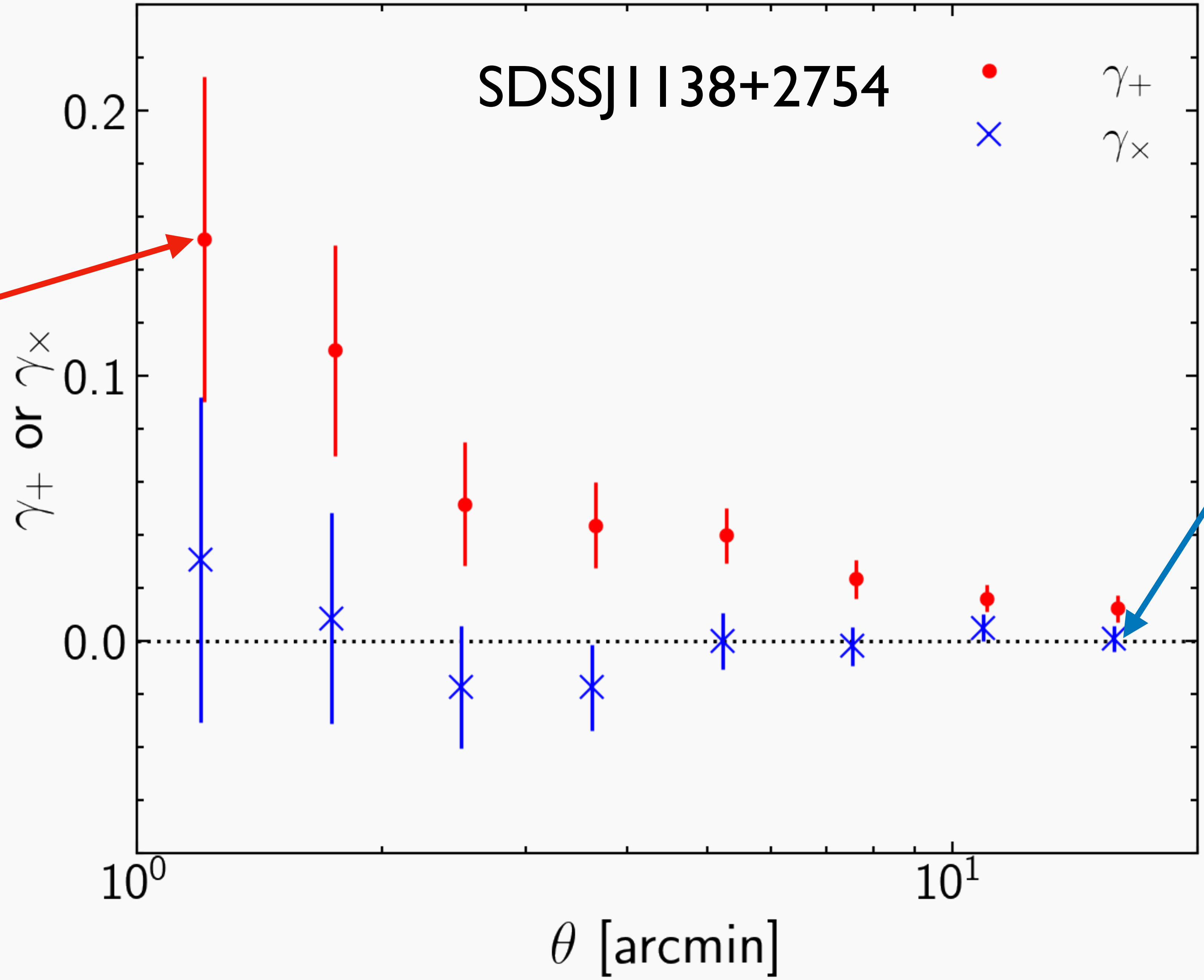
galaxies used for weak lensing analysis



compute tangential shear in each annulus



Tangential and cross shear profiles



tangential shear significantly detected

cross shear consistent with zero as expected

(distance from cluster center)

Extracting information

- we can extract information on the cluster by fitting the observed shear profile with a model
- as examples, we consider **SIS** and **NFW** profiles

Singular Isothermal Sphere (SIS)

- three-dimensional density profile

$$\rho(r) = \frac{\sigma_v^2}{2\pi G r^2}$$

σ_v: velocity dispersion

- convergence and tangential shear profiles

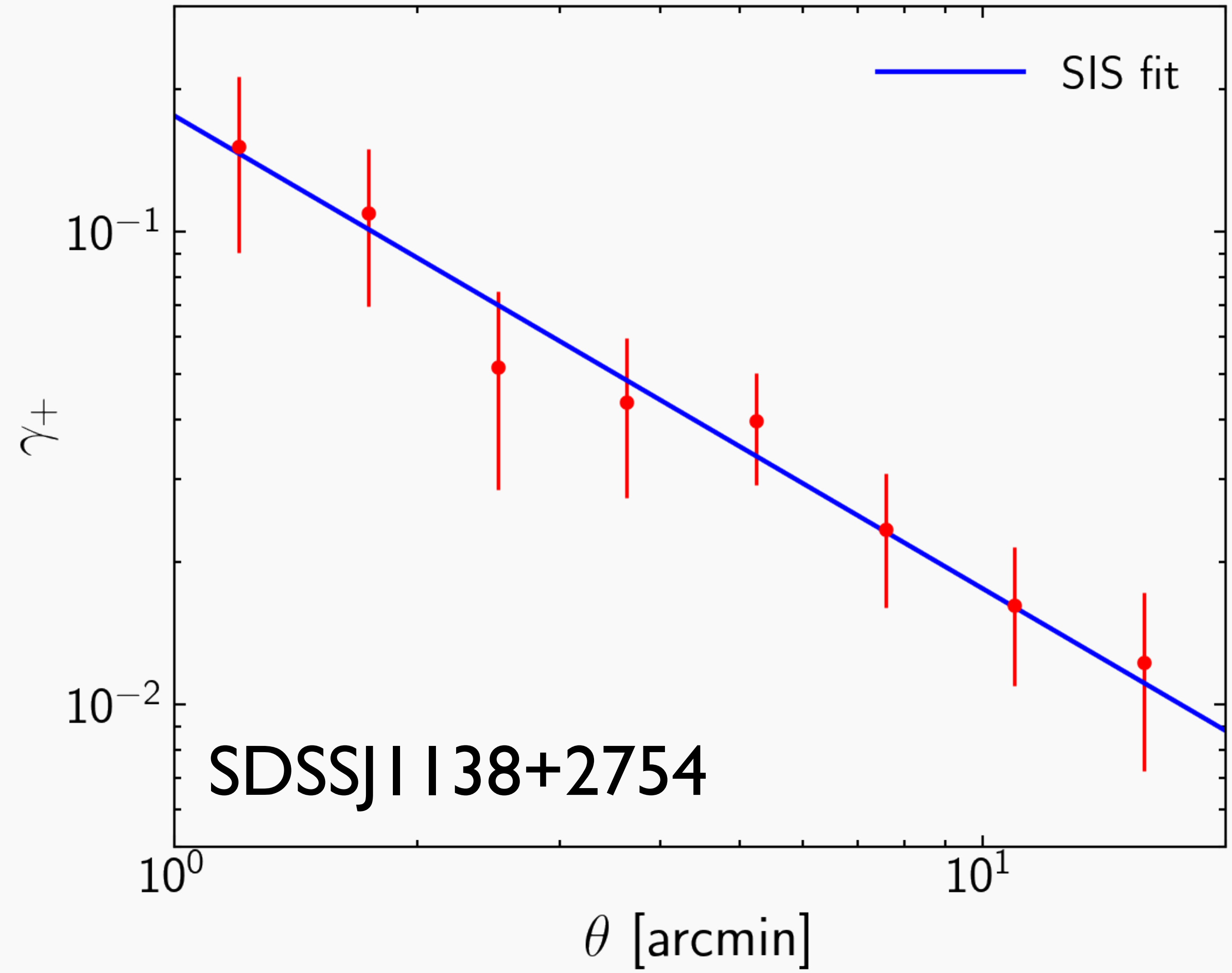
$$\kappa(\theta) = \gamma_+(\theta) = \frac{\theta_{\rm Ein}}{2\theta}$$

$$\theta_{\rm Ein} = 4\pi \left(\frac{\sigma_v}{c}\right)^2 \frac{D_{\rm ls}}{D_{\rm os}}$$

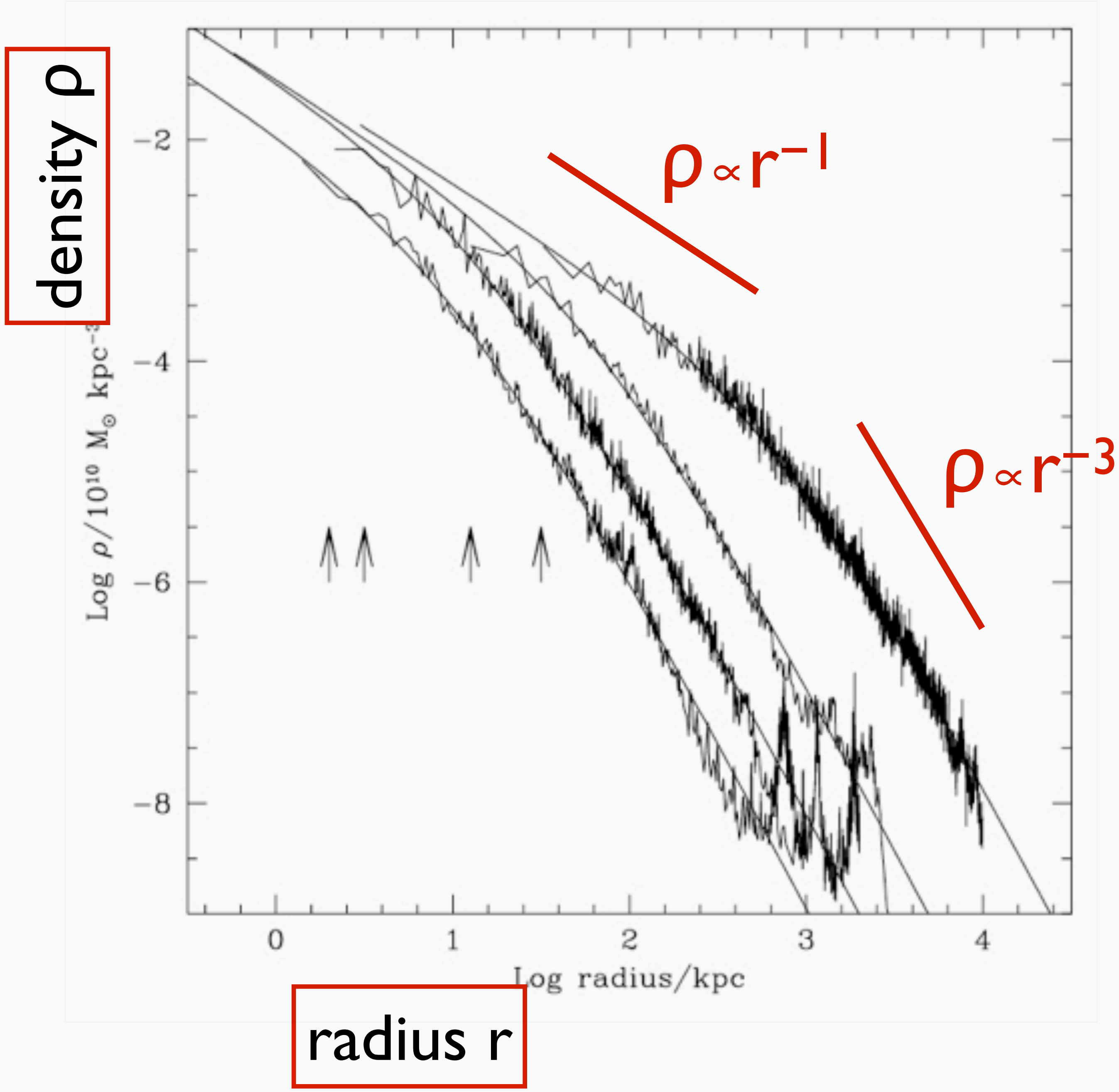
$\theta_{\rm Ein}$: Einstein radius

SIS fitting result

- assuming $\langle z_s \rangle \sim 1$, velocity dispersion is derived to **$\sigma_v \sim 1200$ km/s**
- this corresponds to cluster mass of $M > 10^{15}\ h^{-1} M_\odot$



Navarro-Frenk-White (NFW)



Navarro, Frenk & White (1996, 1997)

- density profile of dark matter halos in N-body simulations

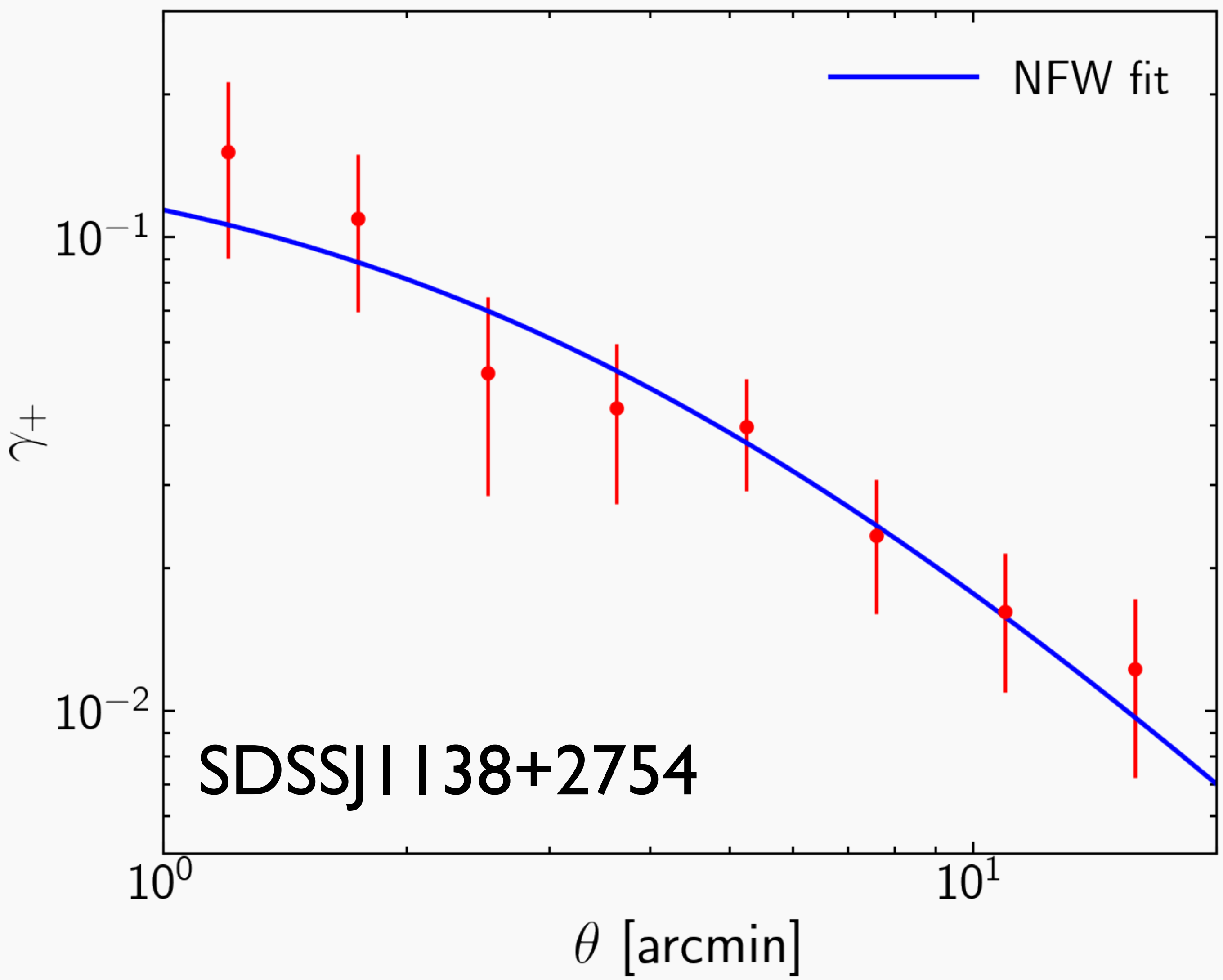
$$\rho(r) = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2}$$

- analytic expression of tangential shear profile available

(e.g., Wright & Brainerd 2000)

NFW fitting result

- good fit achieved
- inferred cluster mass from the fit is $M \sim 10^{15}\ h^{-1} M_\odot$



Example of analysis: summary

- tangential shear profile can be measured for each massive cluster
- by fitting observed profile with a model, we can extract information on dark matter distribution such as total mass

Weak lensing mass map

- tangential shear profile analysis assumed center of the lens and density profile used for fitting
- in fact `mass reconstruction' without any assumption is possible from weak lensing shear data (Kaiser & Squires 1993)


Mass reconstruction

- recap: relation of convergence and shear

$$\gamma_1(\vec{\theta}) = \frac{1}{2}\left(\frac{\partial \alpha_1}{\partial \theta_1} - \frac{\partial \alpha_2}{\partial \theta_2}\right) = \frac{1}{\pi}\int d\vec{\theta}'\,\kappa(\vec{\theta}')\,\frac{(\theta_2 - \theta_2')^2 - (\theta_1 - \theta_1')^2}{\left\{(\theta_1 - \theta_1')^2 + (\theta_2 - \theta_2')^2\right\}^2}$$

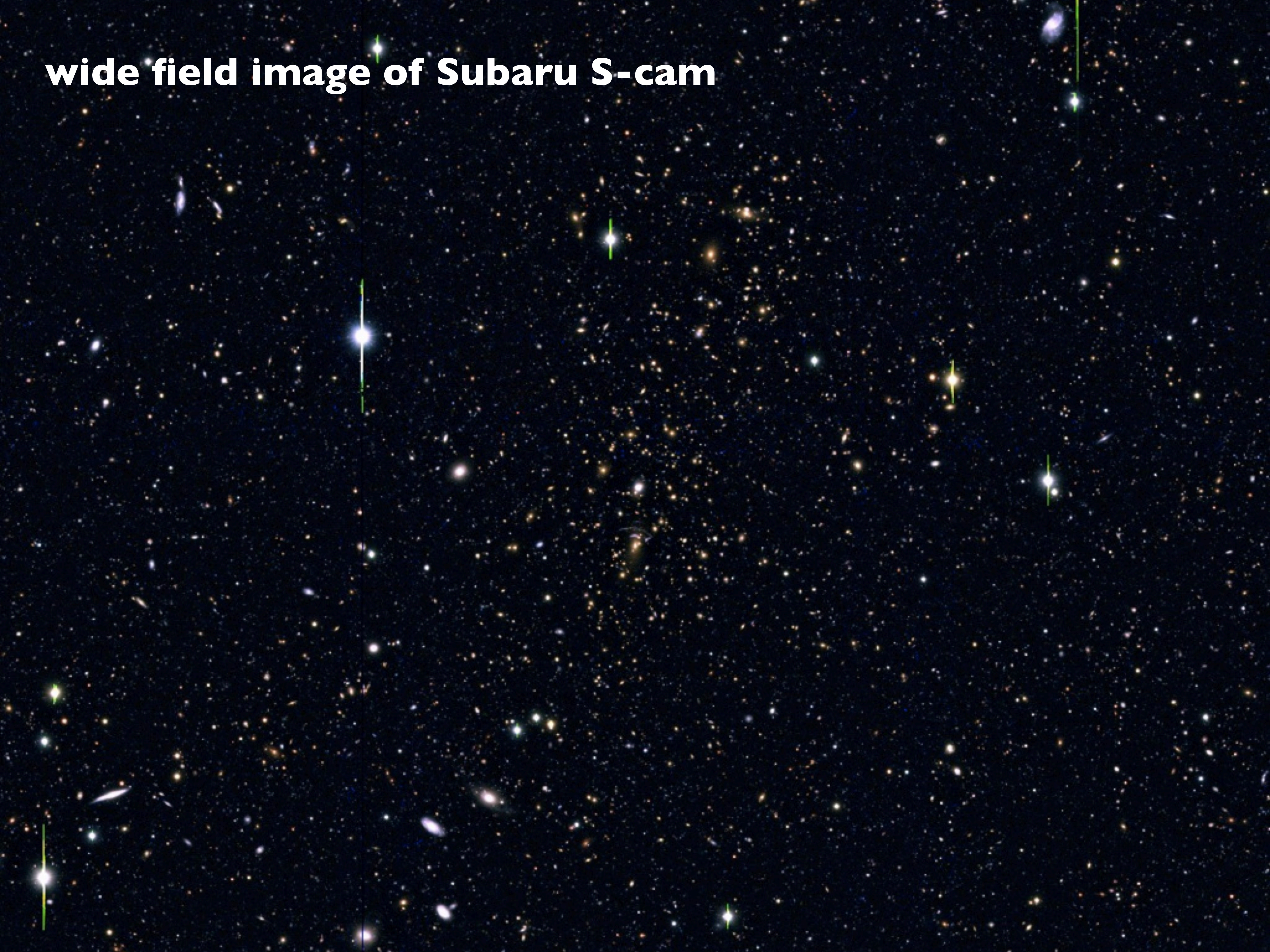
$$\gamma_2(\vec{\theta}) = \frac{\partial \alpha_1}{\partial \theta_2} = \frac{\partial \alpha_2}{\partial \theta_1} = \frac{1}{\pi}\int d\vec{\theta}'\,\kappa(\vec{\theta}')\,\frac{-2(\theta_1 - \theta_1')(\theta_2 - \theta_2')}{\left\{(\theta_1 - \theta_1')^2 + (\theta_2 - \theta_2')^2\right\}^2}$$

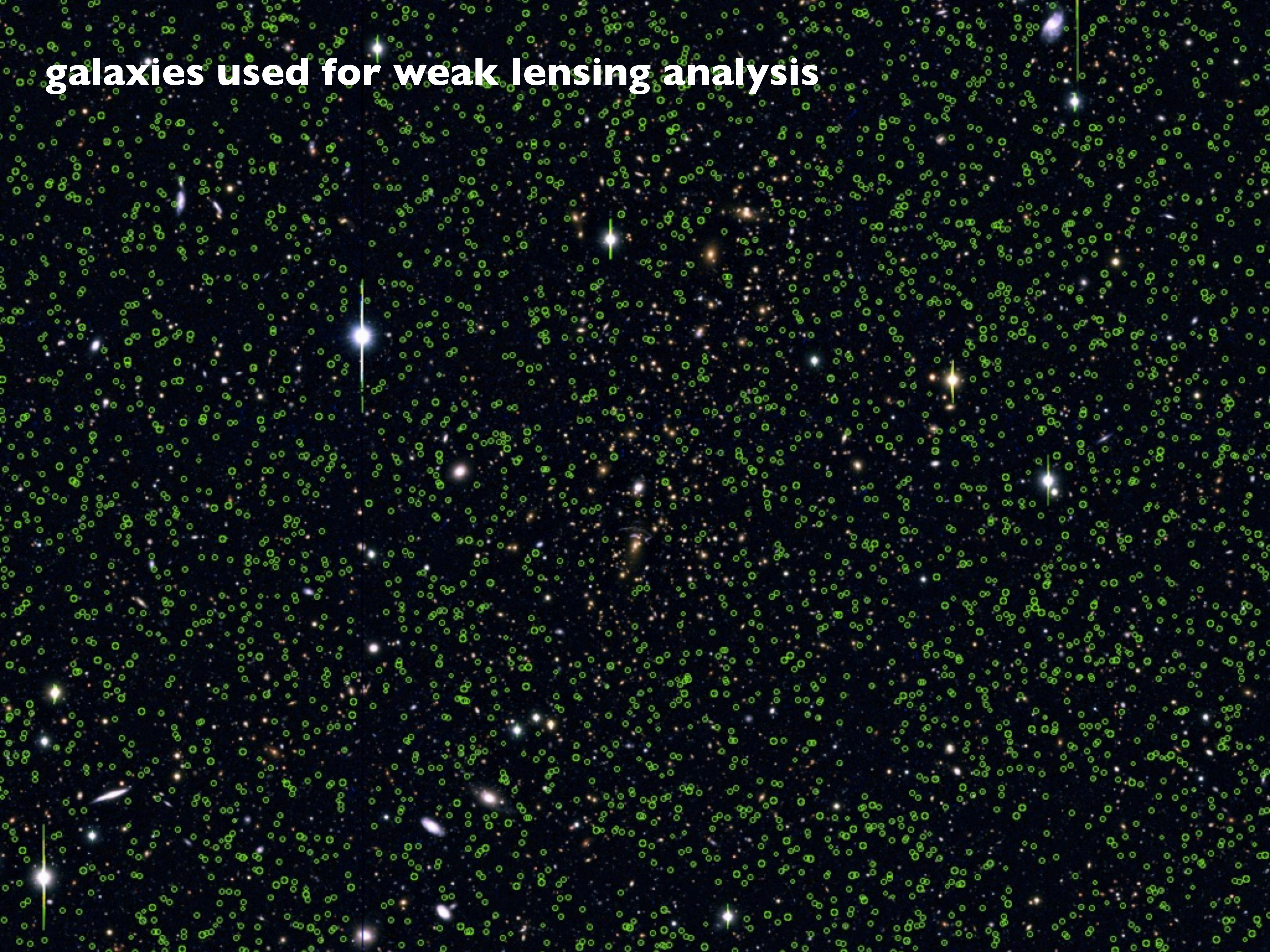
- indicating that convergence is obtained by

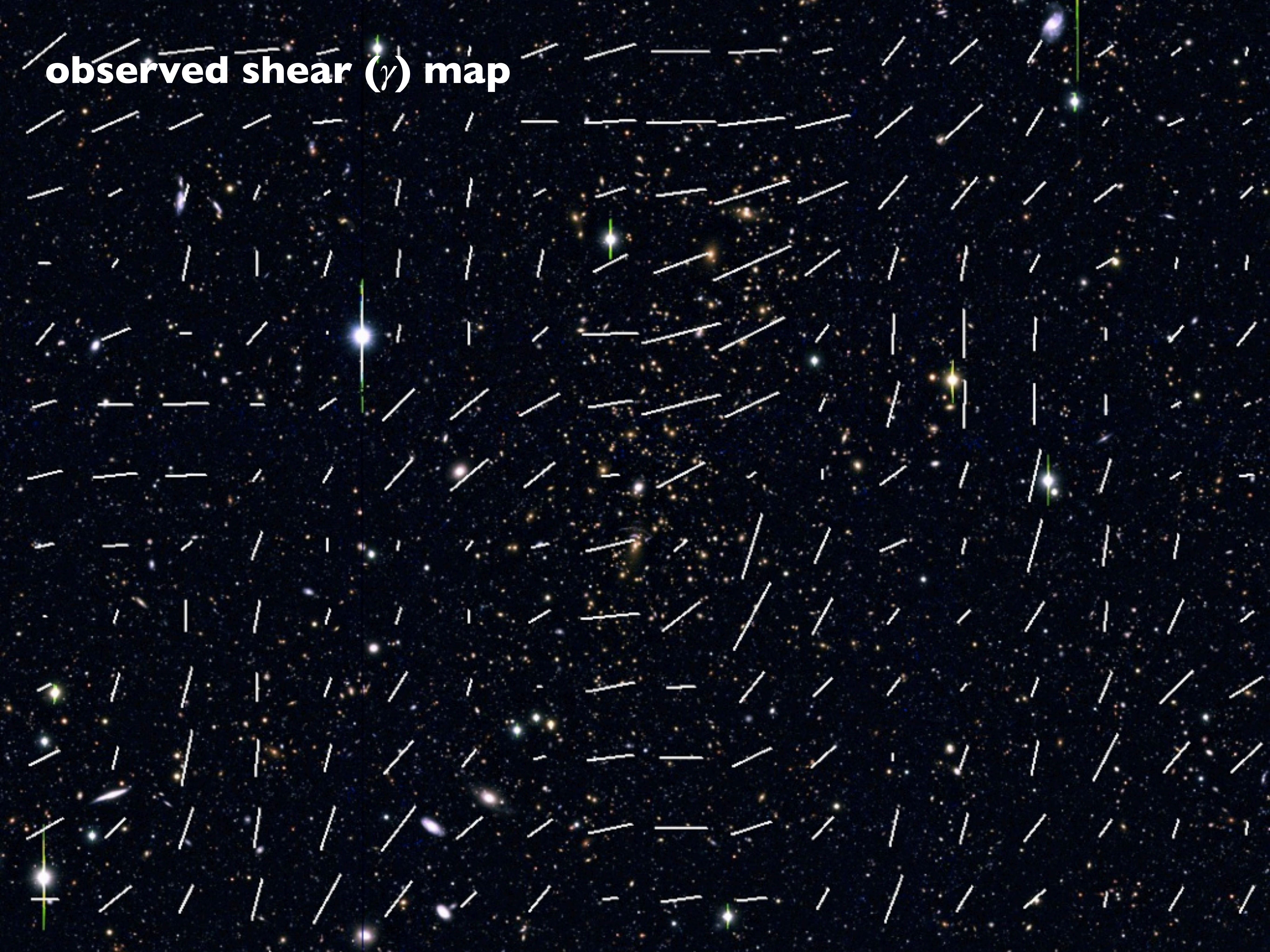
$$\kappa(\vec{\theta}) = \frac{1}{\pi}\int d\vec{\theta}'\left\{\gamma_1(\vec{\theta}') + i\gamma_2(\vec{\theta}')\right\} D^*(\vec{\theta} - \vec{\theta}')$$

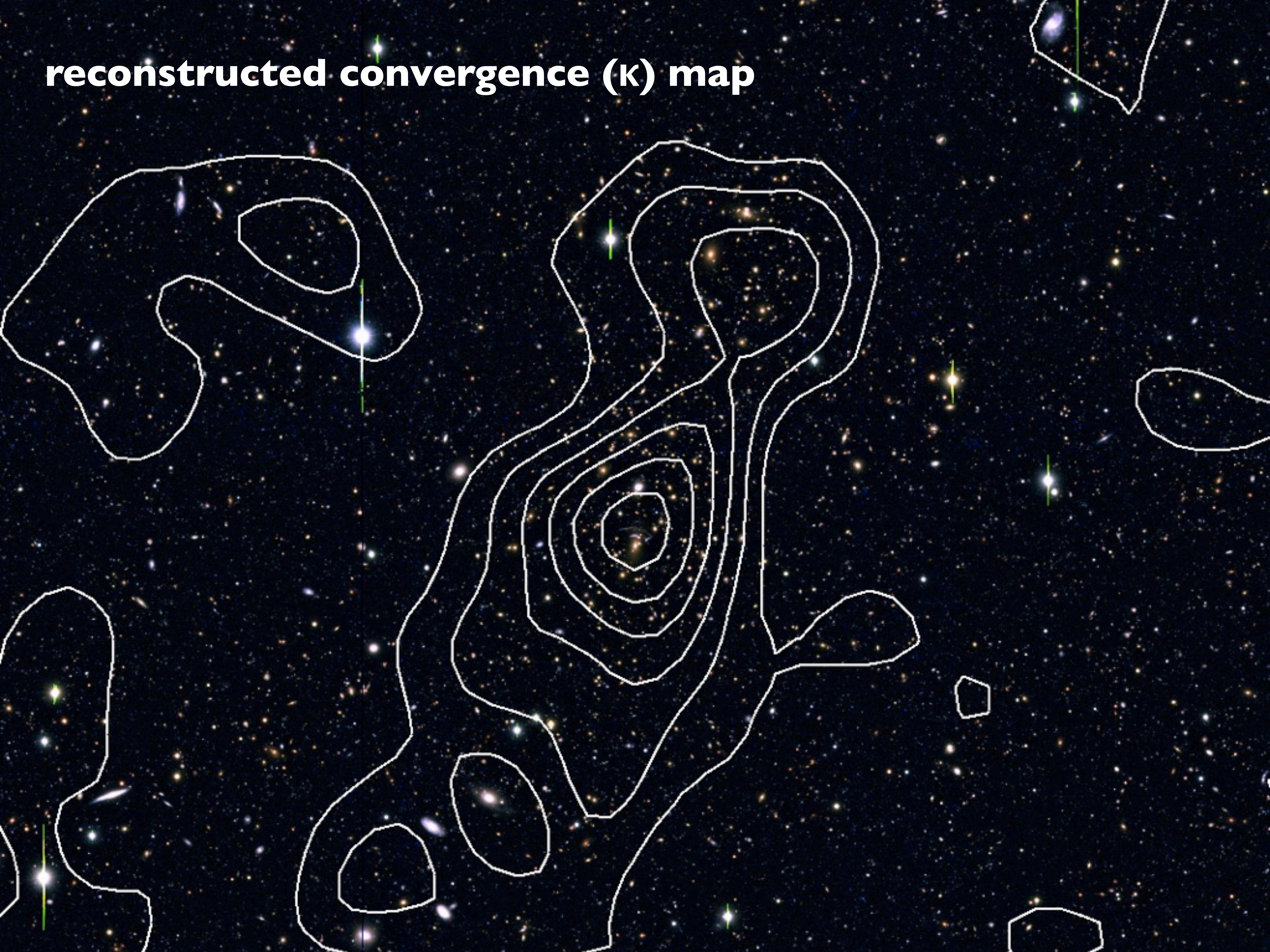
$$D^*(\vec{\theta}) = \frac{\theta_2^2 - \theta_1^2 + 2i\theta_1\theta_2}{|\vec{\theta}|^4}$$

wide field image of Subaru S-cam



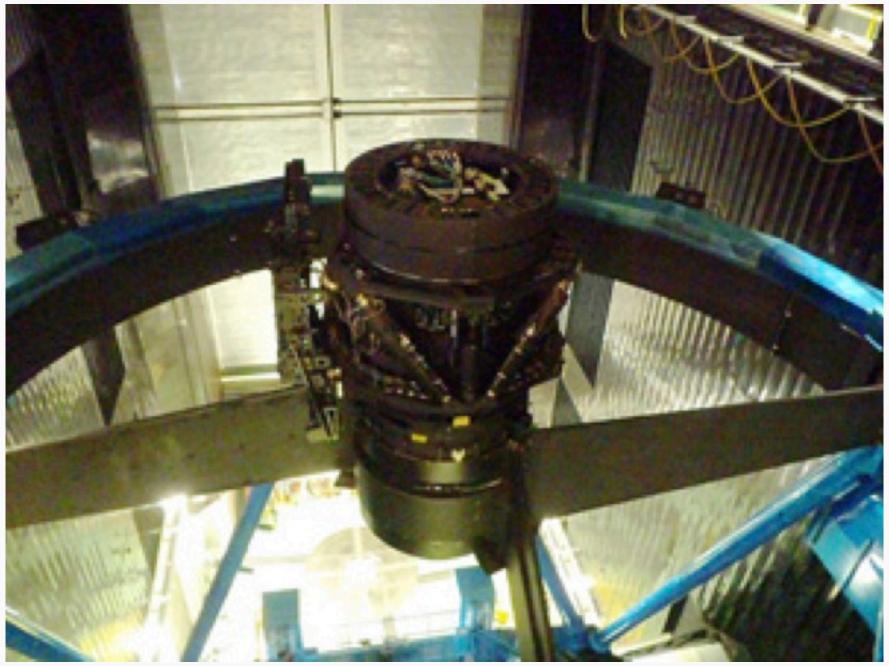

galaxies used for weak lensing analysis

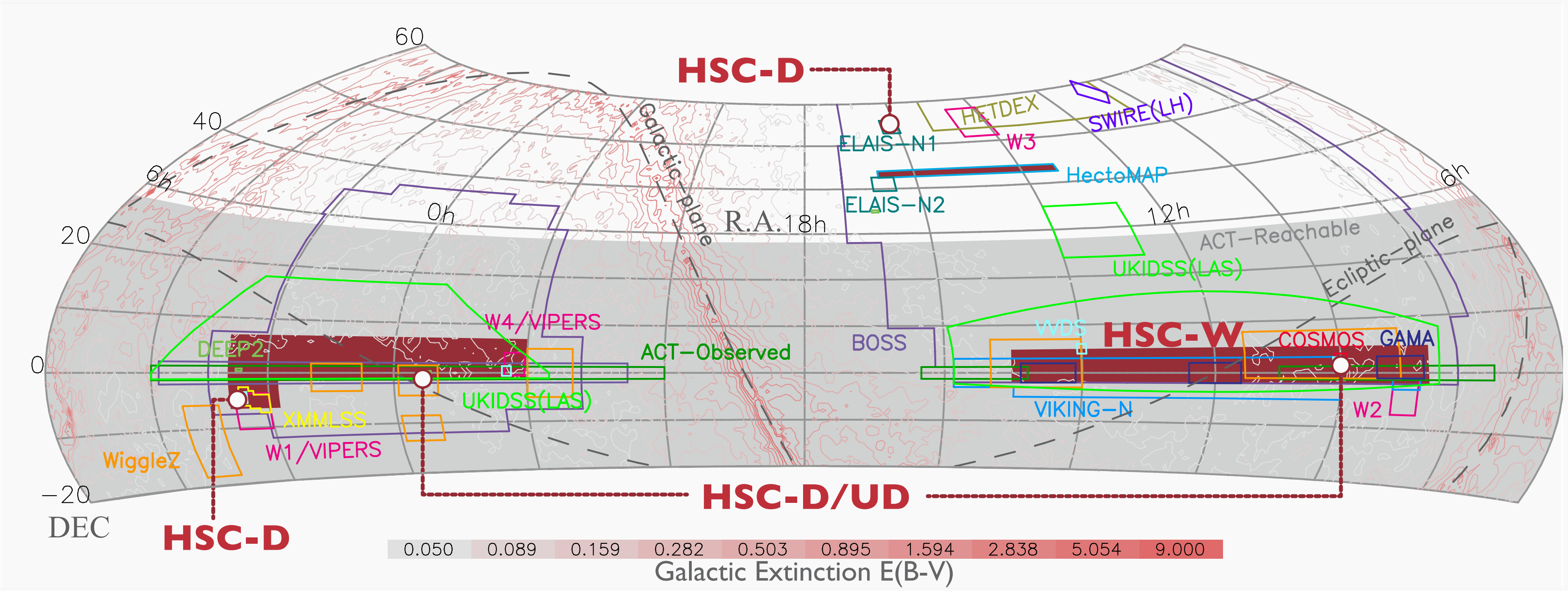

observed shear (γ) map


reconstructed convergence (κ) map

PI: Satoshi Miyazaki (NAOJ)

Hyper Suprime-Cam survey





- a new **wide field** camera mounted on Subaru (**1.7 deg²** covered by **900 million pixels**)
- survey to observe ~1000 deg² of the sky to ~26 mag depth (2014−2021)

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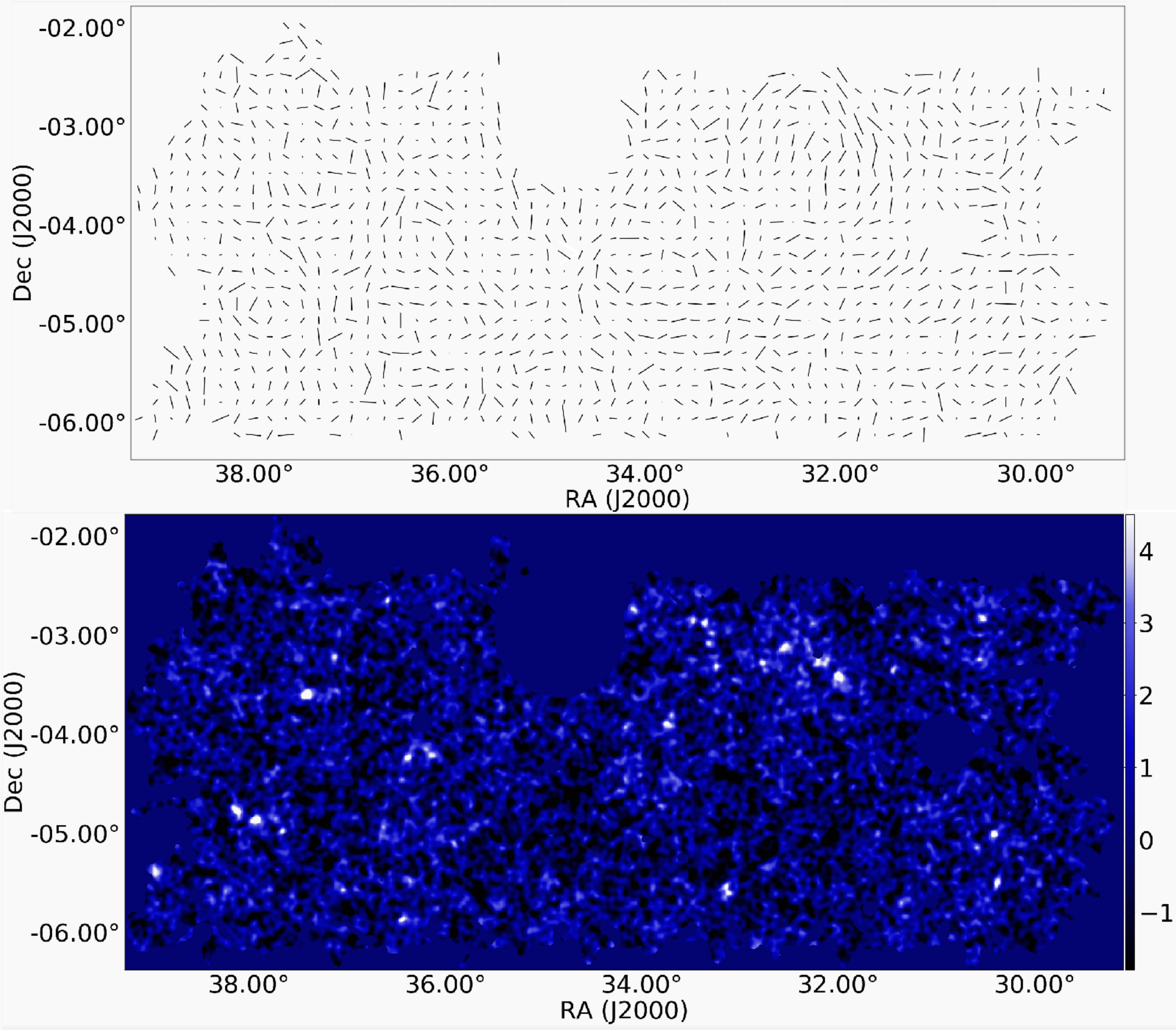
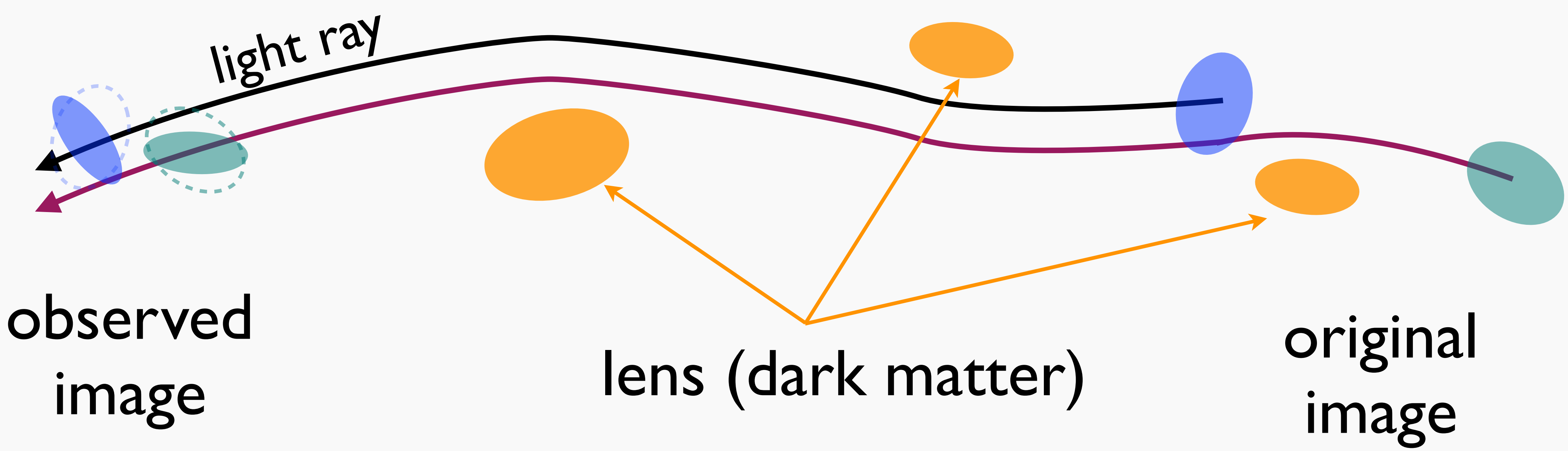
Wide field mass map



coherent lensing distortion (shear)

inferred dark matter distribution (convergence)

3D mass reconstruction



- weak gravitational lensing analysis of galaxies at different distances from us

→ **reconstruction of 3D mass distribution!**

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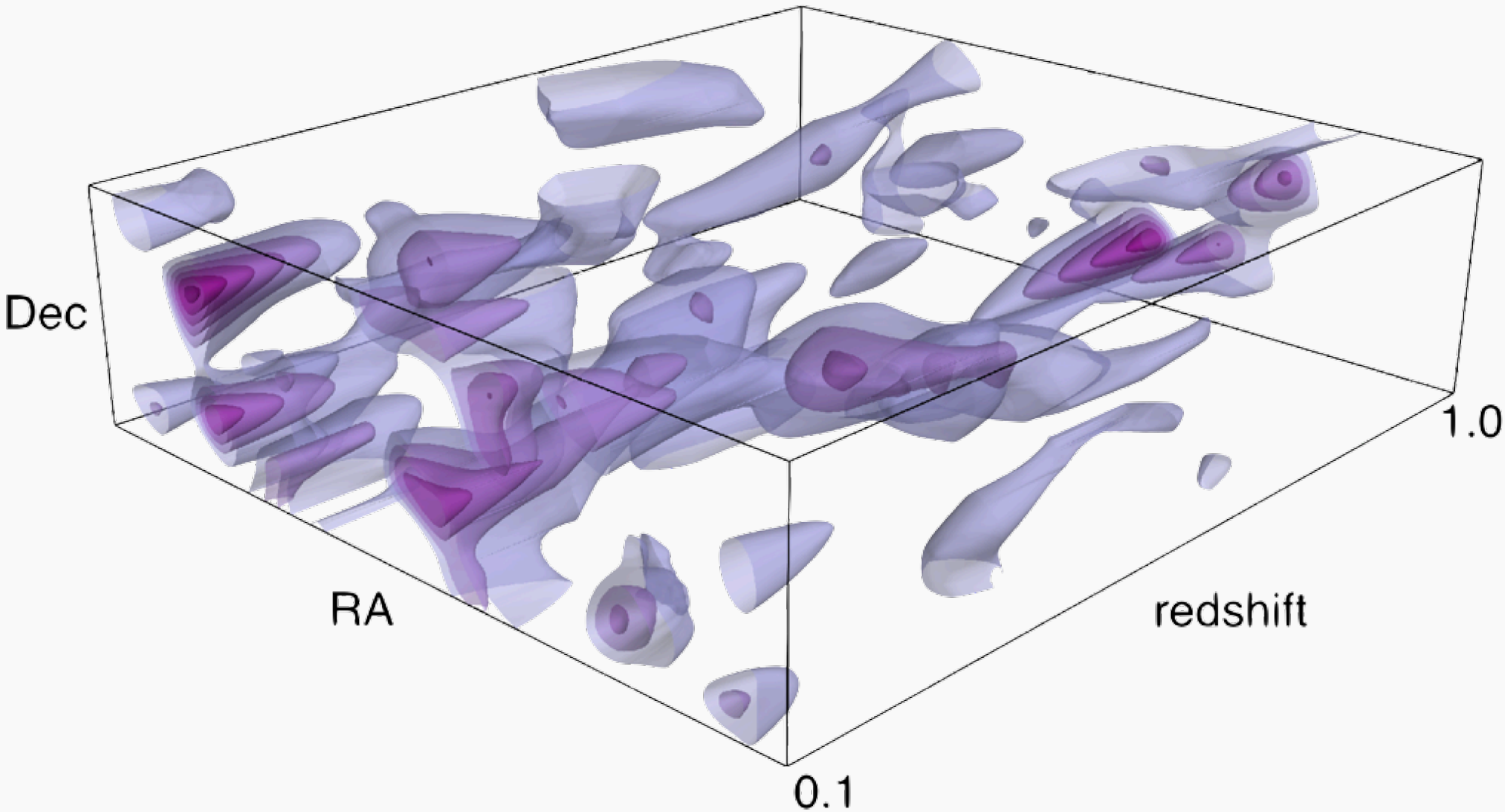
Three-dimensional mass map



***largest three-dimensional dark matter map ever created**

Summary

- weak gravitational lensing provides a powerful means of studying of dark matter distribution
- distortions of background galaxies (**shear**) are related with projected surface mass density (**convergence**)
- we need many galaxies with accurate shape measurements (i.e., wide and deep imaging)