Applications of gravitational lensing in astrophysics and cosmology

3. Weak lensing analysis

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Contents

Strong vs weak lensing

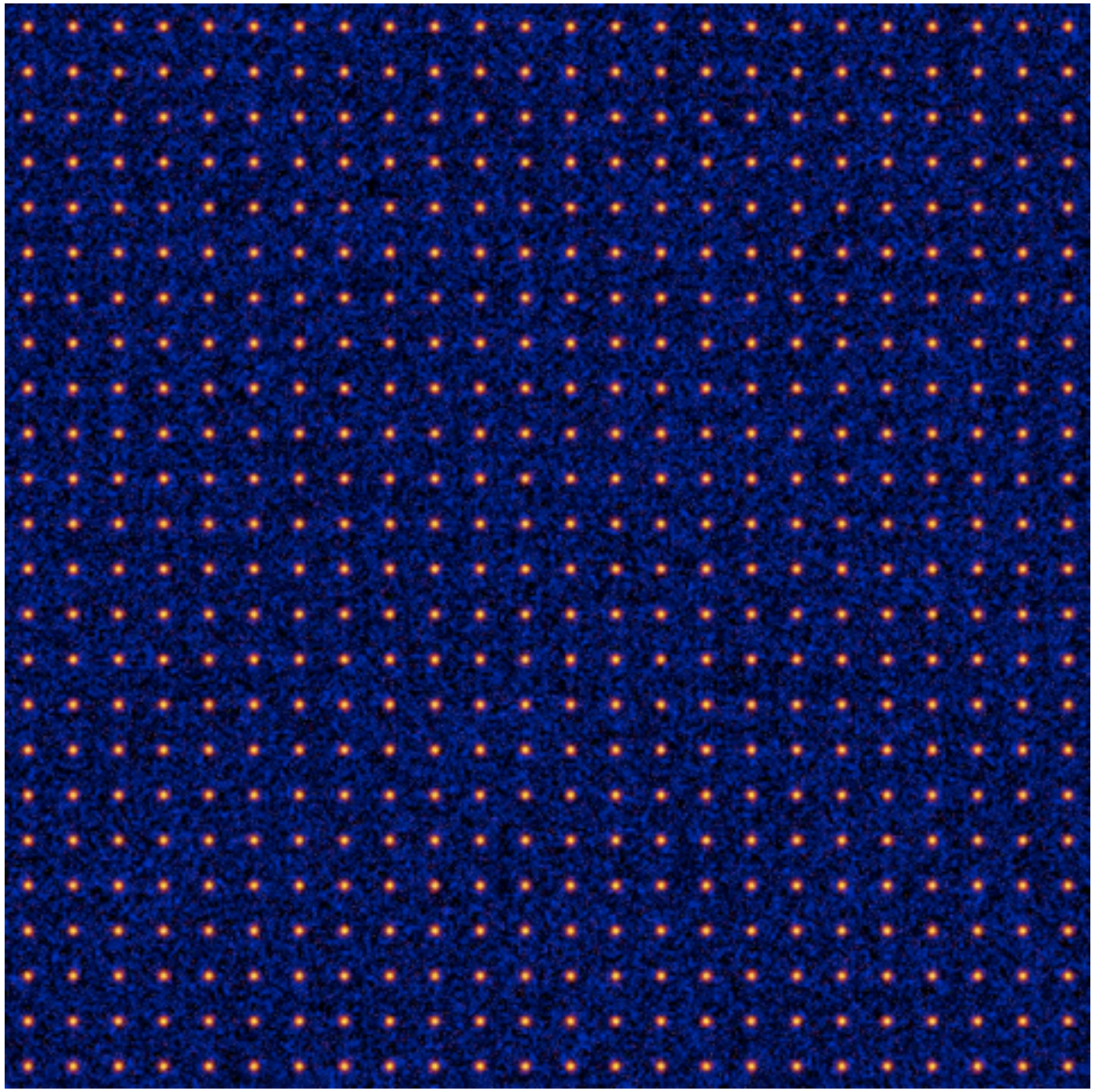
- strong lensing
  - observed for individual sources
  - $\kappa \gtrsim 1$ ($\Sigma \gtrsim \Sigma_{cr}$), near critical curves/caustics
  - multiple images, high elongation/magnification
- weak lensing
  - observed for ensemble of sources
  - $\kappa \ll 1$ ($\Sigma \ll \Sigma_{cr}$), far from critical curves/caustics
  - no multiple image, tiny elongation/magnification

Weak lensing analysis

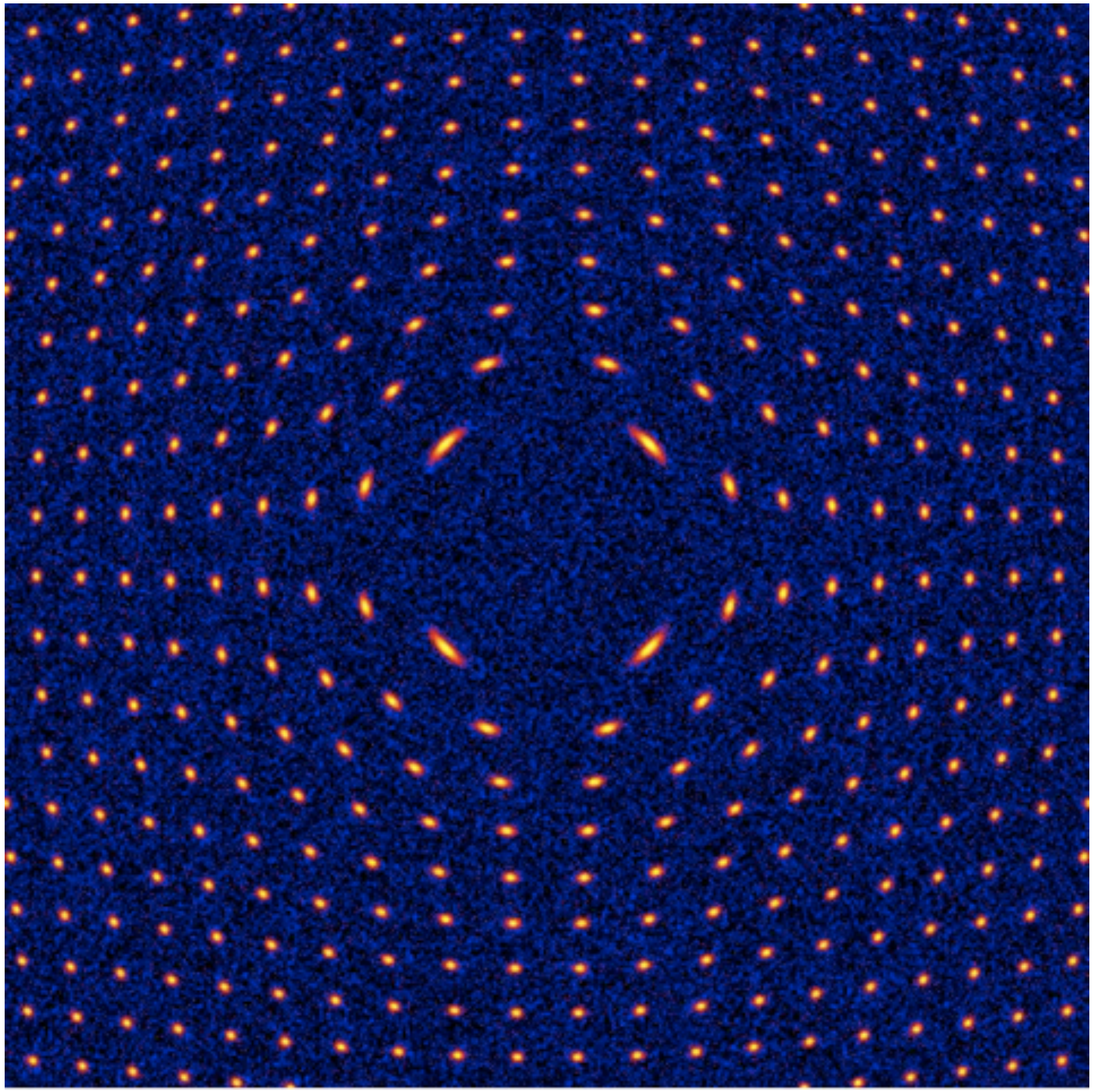
- weak lensing method
- mass reconstruction
- cluster weak lensing
- weak lensing by large-scale structure

simulated by *glafic*

Lensing effect on galaxies



no lensing

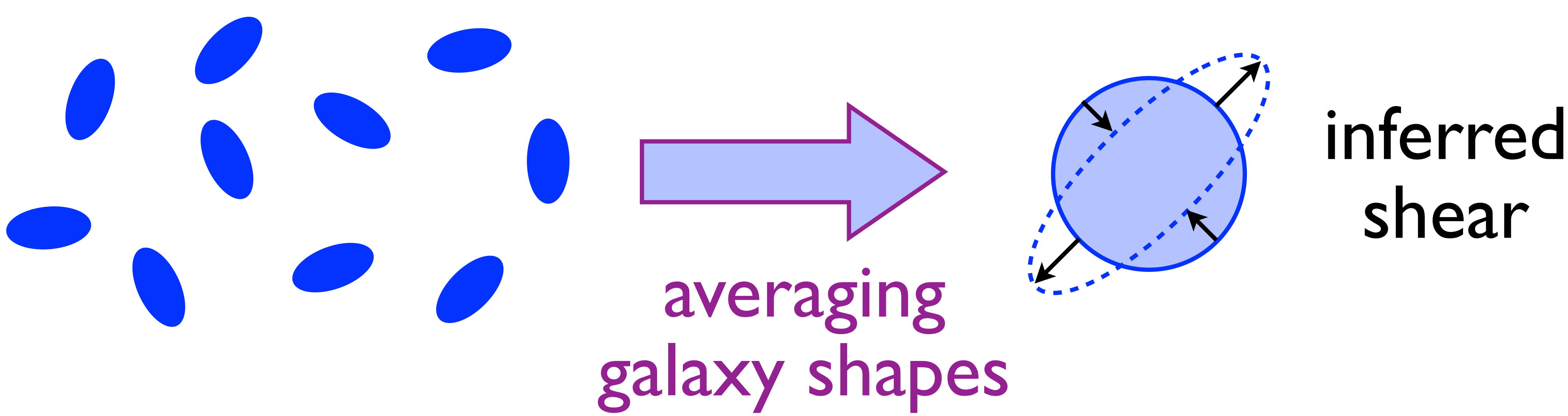


lens potential at the center


simulated by *glafic*

Weak lensing

- lensing distorts background galaxies
- however, each galaxy is not spherical but has intrinsic shape (ellipticity)
- extract lensing distortion by averaging many galaxies' shapes, assuming intrinsic galaxy shapes are randomly oriented



Weak lensing method (I)

- characterize galaxy shapes by moment Q_{ab}

$$Q_{ab} \equiv \frac{\int d\vec{\theta} I(\vec{\theta}) \theta_a \theta_b}{\int d\vec{\theta} I(\vec{\theta})}$$

$I(\vec{\theta})$: galaxy SB profile

- define galaxy 'ellipticity'

$$\epsilon_1 \equiv \frac{Q_{11} - Q_{22}}{Q_{11} + Q_{22}} \qquad \epsilon_2 \equiv \frac{2Q_{12}}{Q_{11} + Q_{22}}$$

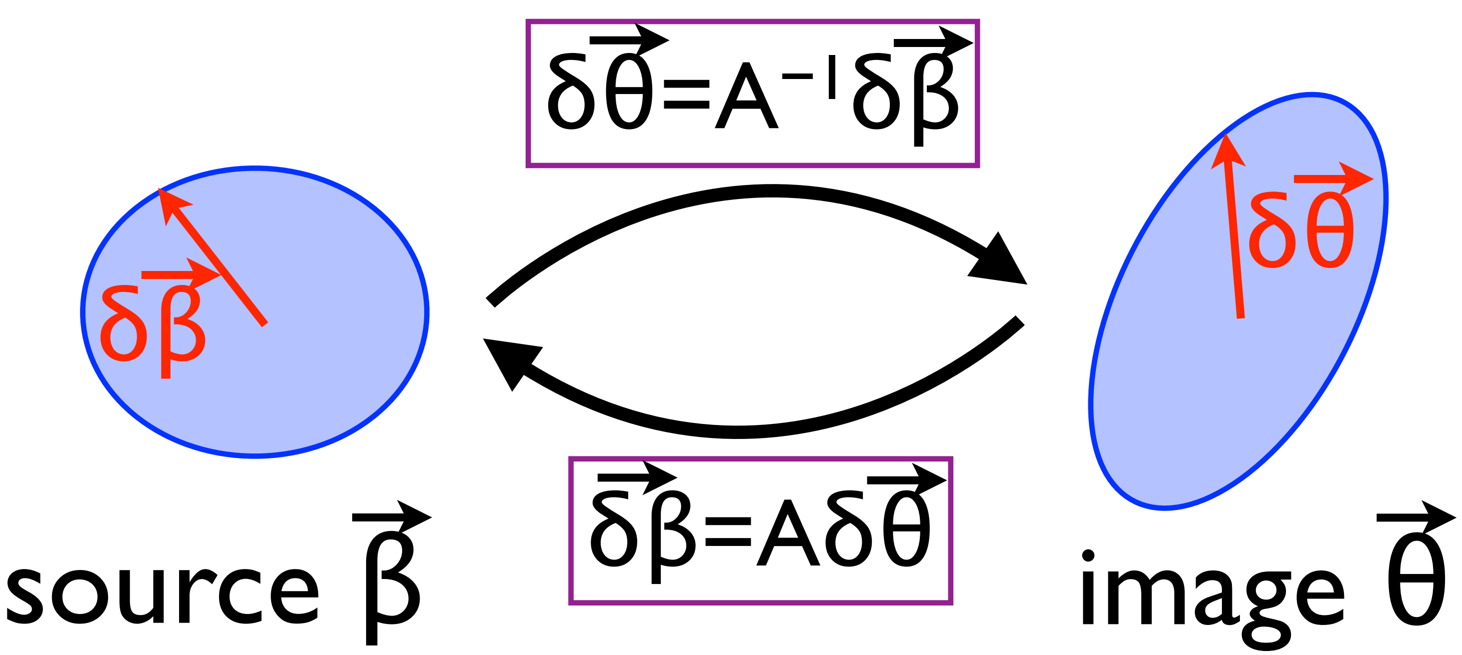
$\epsilon_1 > 0$ $\epsilon_1 < 0$ $\epsilon_2 > 0$ $\epsilon_2 < 0$

Weak lensing method (II)

- lensing change galaxy shape: $Q^{(s)}_{ab} \rightarrow Q_{ab}$

$$Q^{(s)}_{ab} = \frac{\int d\vec{\beta} I^{(s)}(\vec{\beta})\beta_a\beta_b}{\int d\vec{\beta} I^{(s)}(\vec{\beta})} \approx A_{ac}A_{bd}Q_{cd}$$

$$A = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$



A : *de*-lensing
A^{-1}: lensing

Weak lensing method (III)

- therefore, we obtain

$$\epsilon_1^{(s)} \equiv \frac{Q_{11}^{(s)} - Q_{22}^{(s)}}{Q_{11}^{(s)} + Q_{22}^{(s)}} = \frac{(1-\kappa)^2\epsilon_1 - 2(1-\kappa)\gamma_1 + (\gamma_1^2 - \gamma_2^2)\epsilon_1 + 2\gamma_1\gamma_2\epsilon_2}{(1-\kappa)^2 + |\gamma|^2 - 2(1-\kappa)(\gamma_1\epsilon_1 + \gamma_2\epsilon_2)}$$

$$\epsilon_2^{(s)} \equiv \frac{2Q_{12}^{(s)}}{Q_{11}^{(s)} + Q_{22}^{(s)}} = \frac{(1-\kappa)^2\epsilon_2 - 2(1-\kappa)\gamma_2 + (\gamma_2^2 - \gamma_1^2)\epsilon_2 + 2\gamma_1\gamma_2\epsilon_1}{(1-\kappa)^2 + |\gamma|^2 - 2(1-\kappa)(\gamma_1\epsilon_1 + \gamma_2\epsilon_2)}$$

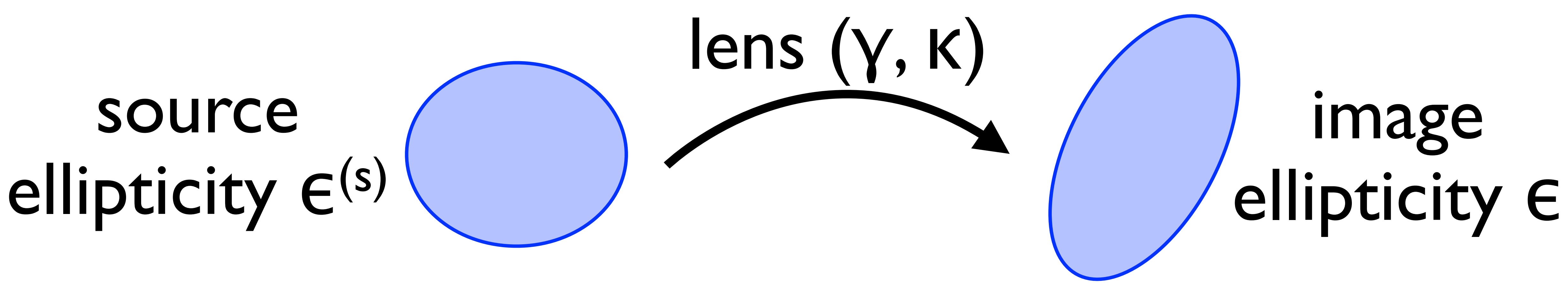
Weak lensing method (IV)

- here we introduce complex shear/ellipticity

$$\gamma \equiv \gamma_1 + i\gamma_2 \qquad \epsilon \equiv \epsilon_1 + i\epsilon_2$$

(γ and ϵ are spin-2 field, i.e., $\gamma \rightarrow \gamma e^{2i\phi}$ under rotation ϕ)

$$\epsilon^{(s)} = \frac{(1-\kappa)^2\epsilon - 2(1-\kappa)\gamma + \gamma^2\epsilon^*}{(1-\kappa)^2 + |\gamma|^2 - 2(1-\kappa)\mathrm{Re}\left[\gamma\epsilon^*\right]}$$



Weak lensing method (V)

- define reduced shear g

$$g \equiv \frac{\gamma}{1-\kappa}$$

- then the equation can be simplified as

$$\boxed{\epsilon^{(s)} = \frac{\epsilon - 2g + g^2\epsilon^*}{1 + |g|^2 - 2\mathrm{Re}\left[g\epsilon^*\right]}}$$

(weak lensing measures g, not γ!)

Weak lensing method (VI)

- random orientation → $\langle \epsilon^{(s)} \rangle = 0$
  + weak shear ($g \ll 1$), $\epsilon \ll 1$

→ $$\langle \epsilon \rangle = 2g$$

- error on estimated reduced shear g is

$$\sigma_g = \frac{\sigma_\epsilon}{2\sqrt{N_{\rm gal}}}$$

$\sigma_\epsilon \sim 0.4$: error on intrinsic ellipticity
N_{gal} : number of galaxies

cluster: $g \sim 0.03$ → $N_{gal} \gtrsim 10^4$ for enough S/N
cosmic shear: $g \sim 0.003$ → $N_{gal} \gtrsim 10^6$ for enough S/N

Shear to mass distribution

(1) assume a model (e.g., NFW), compute shear, compare with observations to constrain parameters to get $\kappa(\vec{\theta})$

(2) mass reconstruction techniques to directly obtain κ-map from shear (e.g., Kaiser & Squires 1993)

Mass reconstruction (I)

- recall: lens potential ψ and convergence κ

$$\psi(\vec{\theta}) = \frac{1}{\pi}\int d\vec{\theta}'\kappa(\vec{\theta}')\ln\left|\vec{\theta}-\vec{\theta}'\right|$$

- shear γ and convergence κ are related as

$$\gamma(\vec{\theta}) = \frac{1}{\pi}\int d\vec{\theta}'\kappa(\vec{\theta}')D(\vec{\theta}-\vec{\theta}')$$

$$D(\vec{\theta}) \equiv \frac{\theta_2^2-\theta_1^2-2i\theta_1\theta_2}{|\vec{\theta}|^4}$$

Mass reconstruction (II)

- convolution → product in Fourier space

$$\hat{\gamma}(\vec{\ell}) = \frac{1}{\pi}\hat{\kappa}(\vec{\ell})\hat{D}(\vec{\ell})$$

$$\hat{D}(\vec{\ell}) = \pi\frac{\ell_1^2 - \ell_2^2 + 2i\ell_1\ell_2}{|\vec{\ell}|^2} = \frac{\pi^2}{\hat{D}^*(\vec{\ell})}$$

$$\rightarrow \boxed{\kappa(\vec{\theta}) - \kappa_0 = \frac{1}{\pi}\int d\vec{\theta}'\gamma(\vec{\theta}')D^*(\vec{\theta} - \vec{\theta}')}$$

constant
→ not affect γ

$$D^*(\vec{\theta}) \equiv \frac{\theta_2^2 - \theta_1^2 + 2i\theta_1\theta_2}{|\vec{\theta}|^4}$$

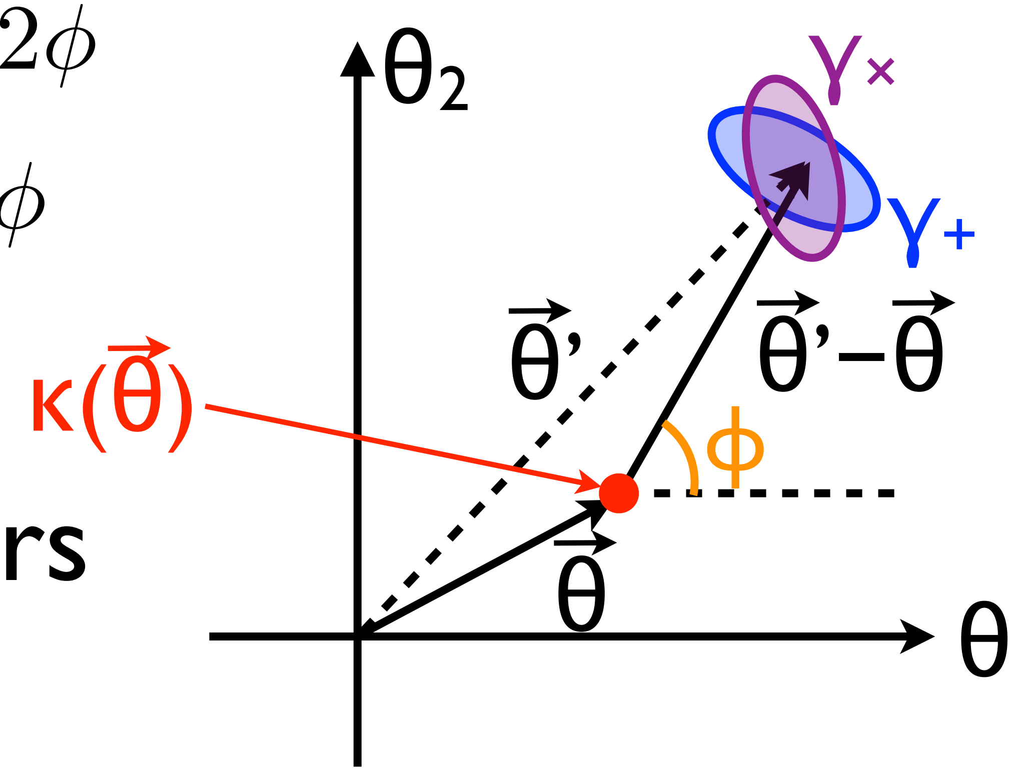
Mass reconstruction (III)

- more explicitly it is written as

$$\kappa(\vec{\theta}) - \kappa_0 = \underbrace{\frac{1}{\pi}\int d\vec{\theta}' \frac{\gamma_+(\vec{\theta}';\vec{\theta})}{|\vec{\theta}-\vec{\theta}'|^2}}_{\text{E-mode (real)}} + i\underbrace{\frac{1}{\pi}\int d\vec{\theta}' \frac{\gamma_\times(\vec{\theta}';\vec{\theta})}{|\vec{\theta}-\vec{\theta}'|^2}}_{\text{B-mode (must vanish)}}$$

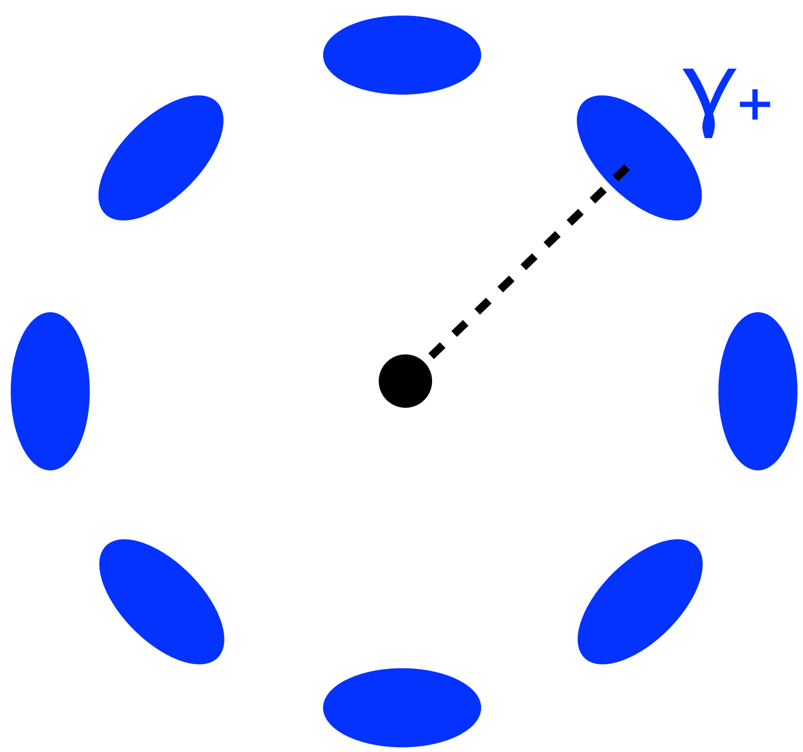
$$\gamma_+(\vec{\theta}';\vec{\theta}) \equiv -\gamma_1 \cos 2\phi - \gamma_2 \sin 2\phi$$

$$\gamma_\times(\vec{\theta}';\vec{\theta}) \equiv \gamma_1 \sin 2\phi - \gamma_2 \cos 2\phi$$

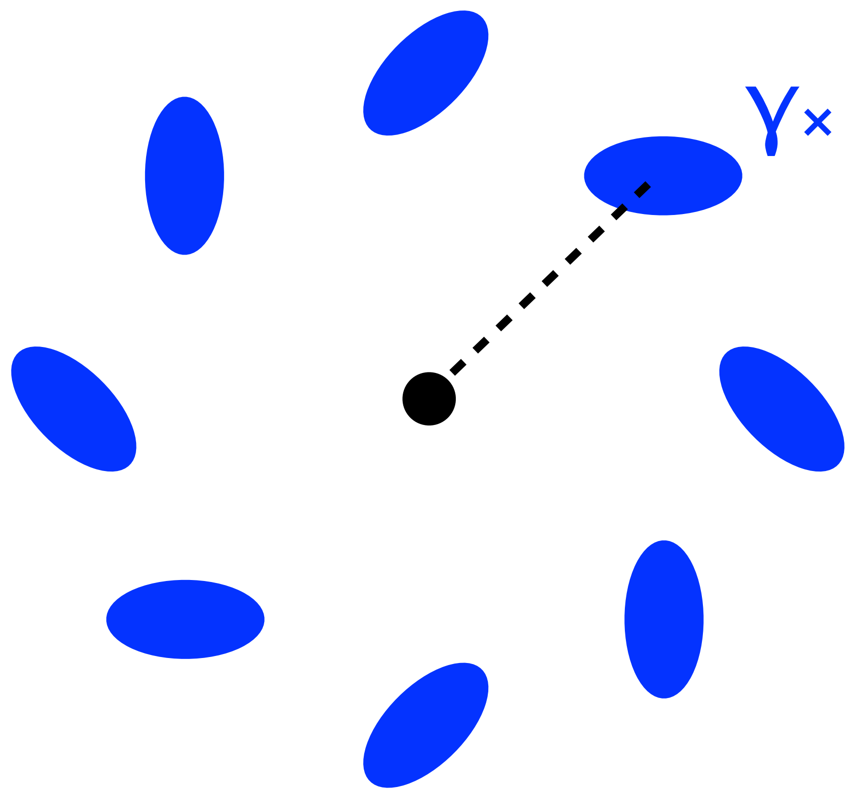


summing up tangential shears
→ convergence

Lensing E-mode/B-mode



"E-mode" generated by gravitational lensing



"B-mode" not generated by gravitational lensing (used to check systematics)

Mass reconstruction (IV)

- in practice, we apply a "filter" to enhance S/N

$$\tilde{\kappa}(\vec{\theta}) = \int d\vec{\theta}\,\kappa(\vec{\theta})U(|\vec{\theta}' - \vec{\theta}|)$$

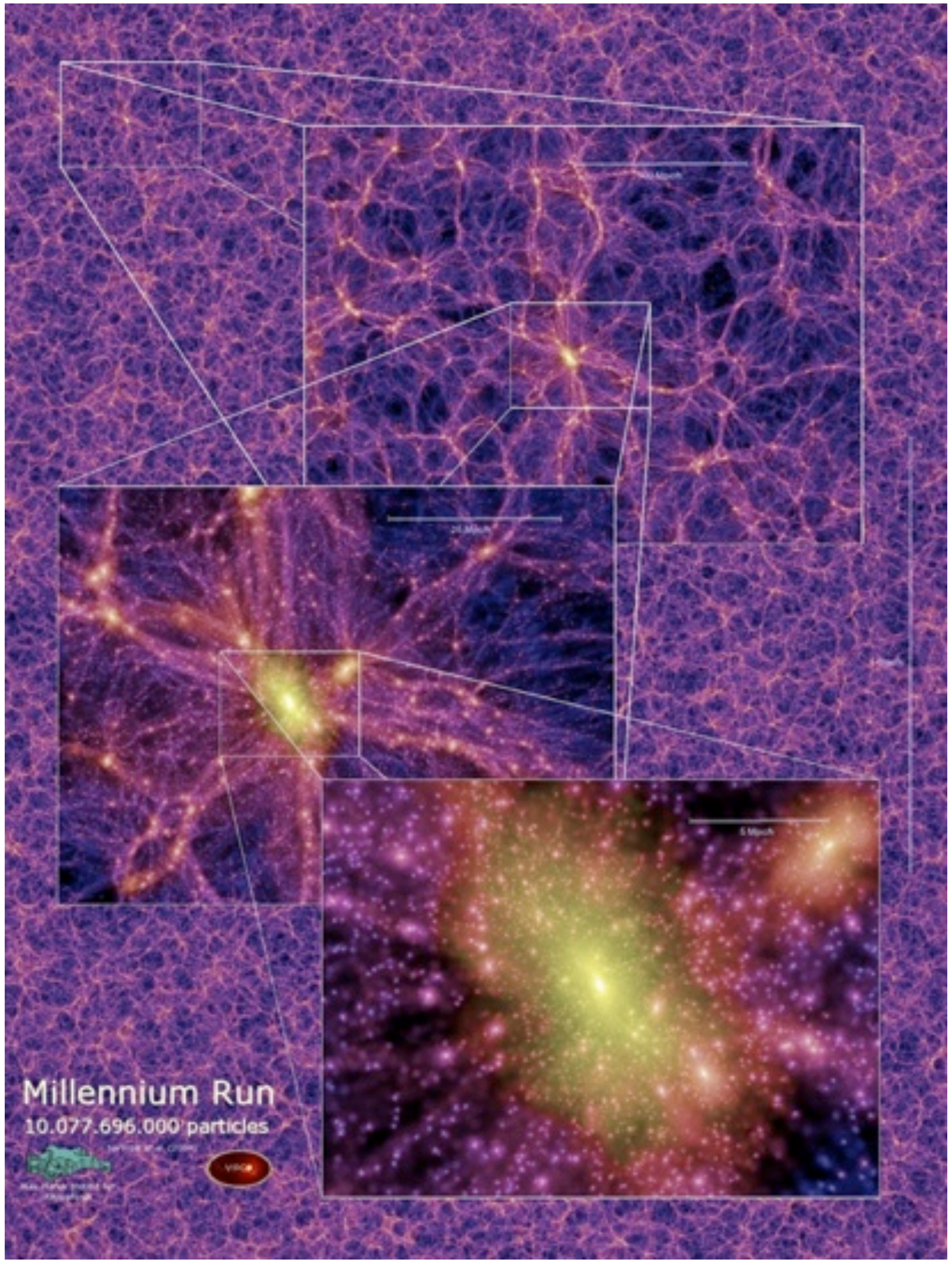
- mass reconstruction w/ filter (Schneider 1996)

$$\tilde{\kappa}(\vec{\theta}) = \int d\vec{\theta}\,\gamma_+(\vec{\theta}';\vec{\theta})Q(|\vec{\theta}' - \vec{\theta}|)$$

$$Q(\theta) = \frac{2}{\theta^2}\int_0^{\theta} d\theta'\theta' U(\theta') - U(\theta)$$

Cluster weak lensing

- cluster of galaxies
  - most massive virialized object in the universe
  - internal structure mostly determined by the dynamics of dark matter
  - useful for studying dark matter and cosmology!



http://www.mpa-garching.mpg.de/galform/millennium/

Cluster profile and weak lensing

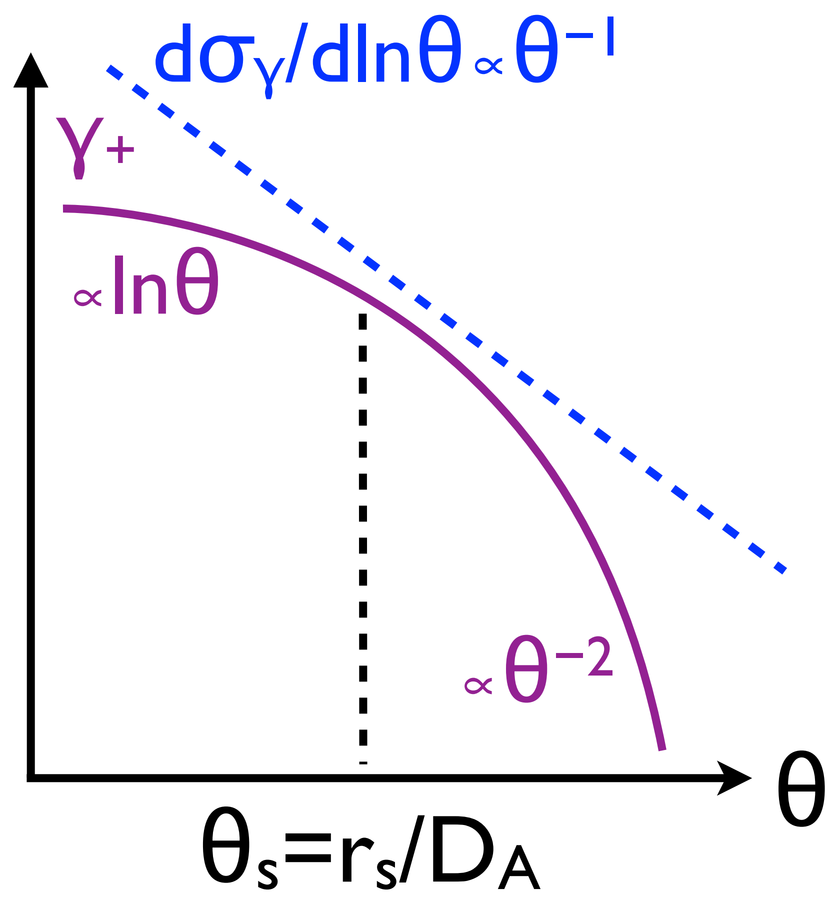
- mass distribution follows NFW profile

$$\rho(r) = \frac{\rho_s}{(r/r_s)(1+r/r_s)^2}$$

- error in each logarithmic bin is

$$\frac{d\sigma_\gamma}{d\ln\theta} \propto \frac{1}{\sqrt{A_{\rm bin}}} \propto \frac{1}{\theta}$$

- therefore S/N is maximum at around $r \approx r_s$



Weak lensing analysis: an example

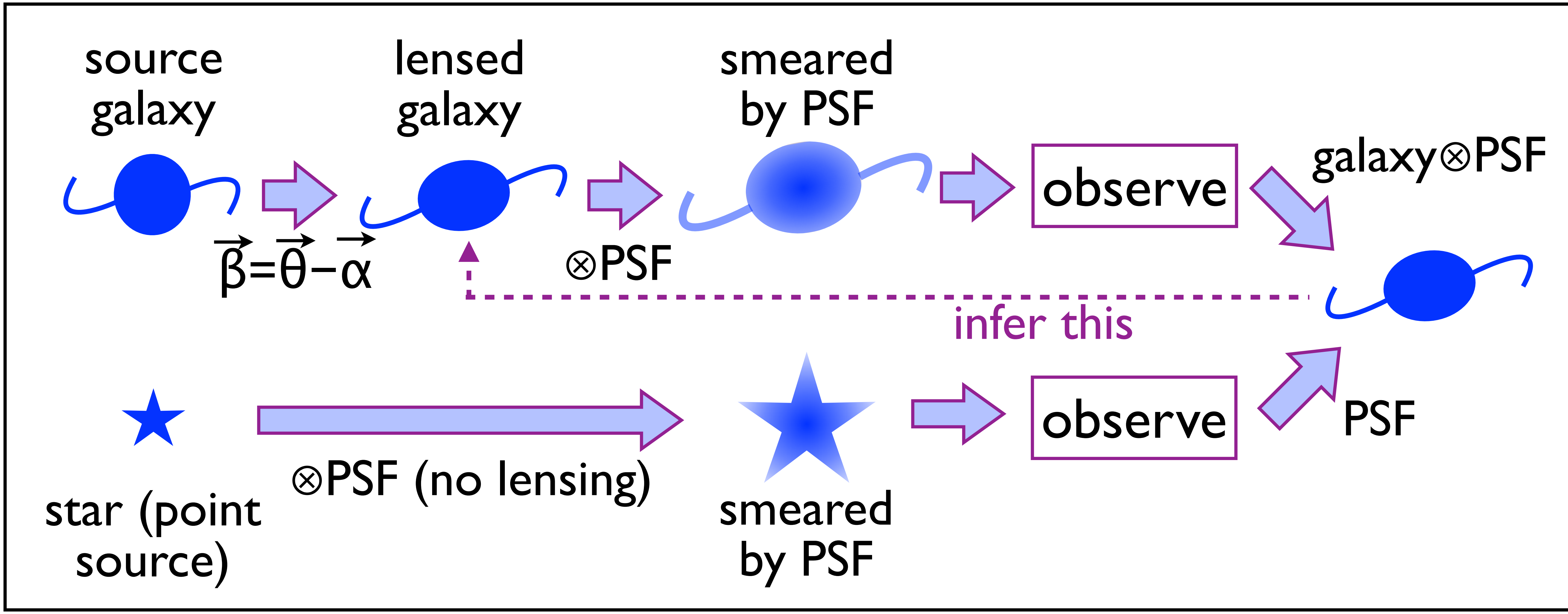
- SDSSJ1138+2754 massive cluster at z=0.45 showing giant arcs, from Sloan Giant Arcs Survey (SGAS)
- Subaru/Suprime-cam gri images for weak lensing analysis

Subaru/Suprime-cam gri-band

Analysis in a real world

- observed galaxy profiles are smeared by Point Spread Function (PSF) from telescope optics, fluctuation of atmosphere, ...
- however we can use images of stars to get information on PSF, and de-convolve PSF
- there are several approaches, including moment-based method (KSB, etc.), model fitting, ...

Concept of analysis



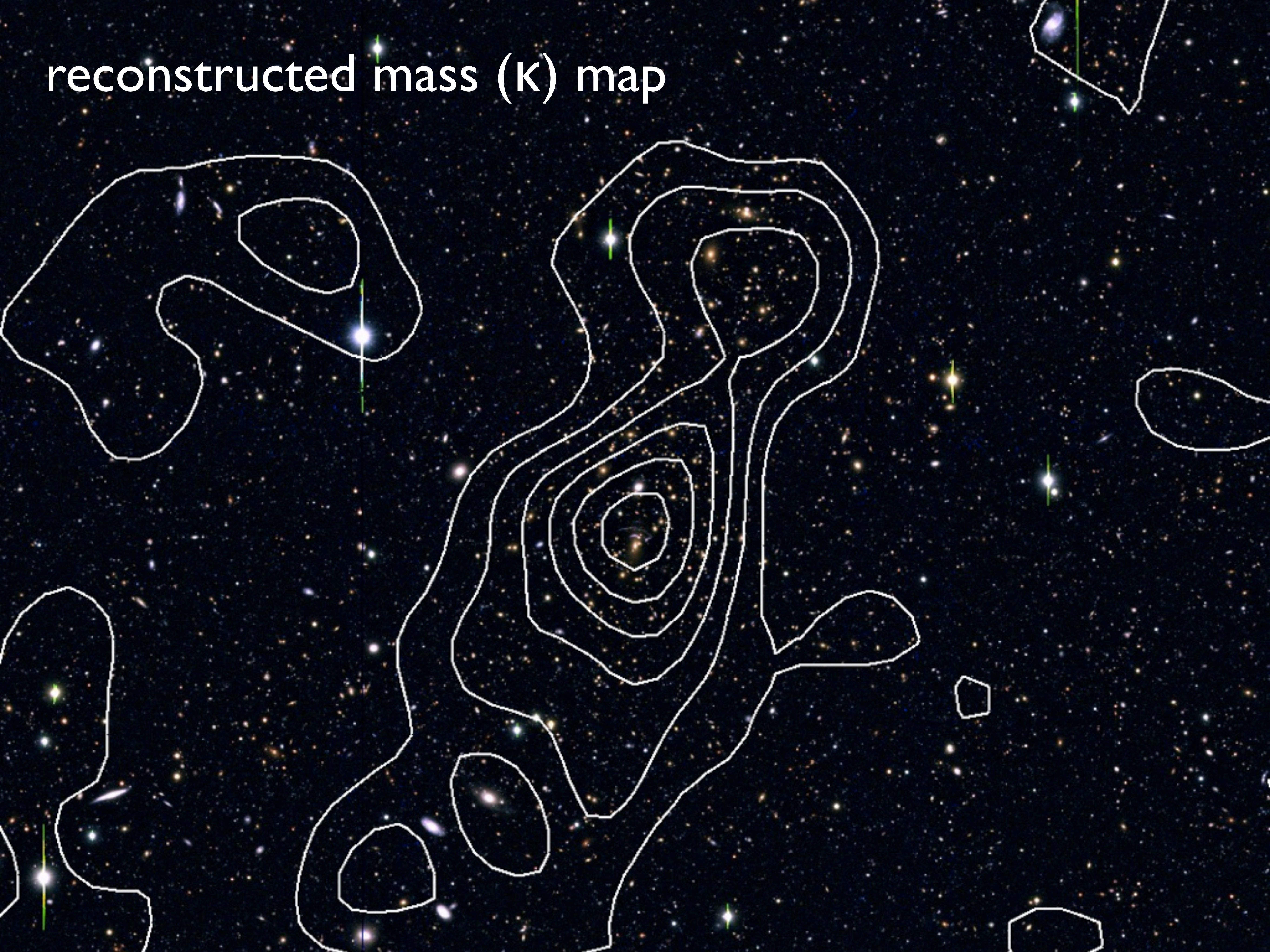
- unbiased shear estimate is one of the biggest challenges in weak lensing analysis
- however, it can fully be checked w/ simulations


wide-field Subaru image

background galaxies used for weak lensing

observed weak lensing shear map

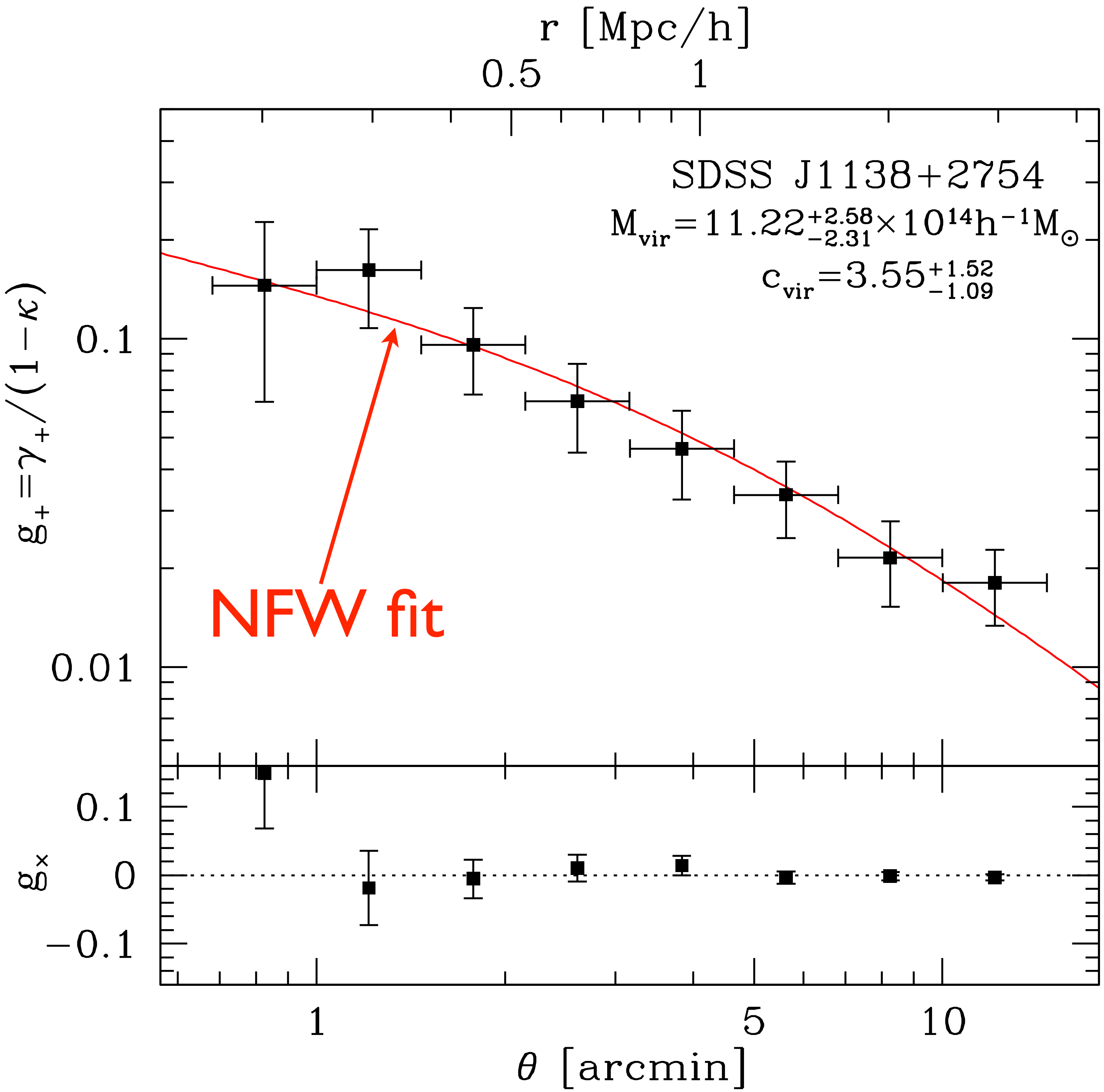
reconstructed mass (κ) map



Oguri et al. MNRAS, **420**, 3213 (2012)

Tangential shear profile

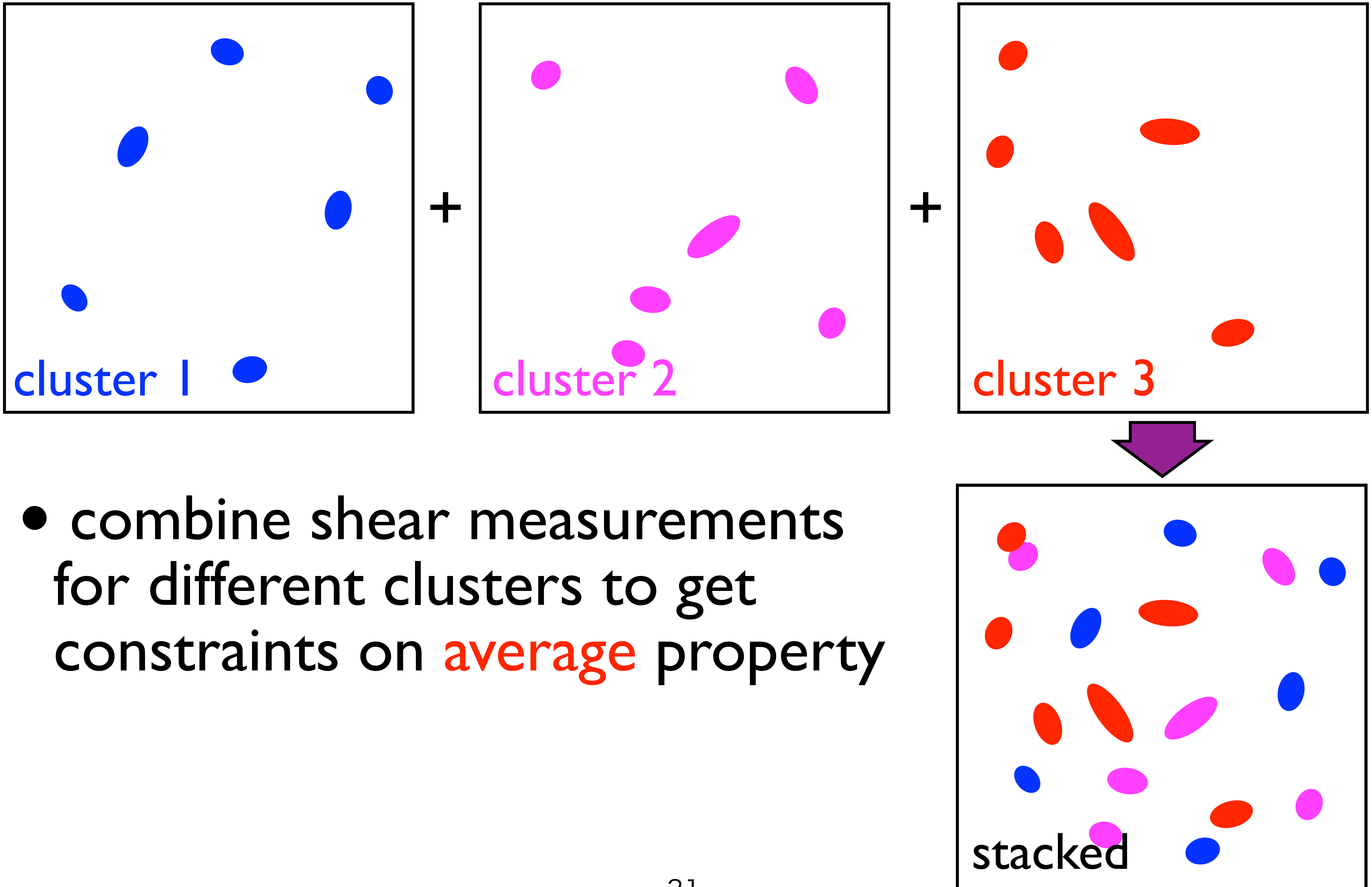
- shear profile very well fitted by the NFW model
- suggest that this cluster is massive, $M_{vir} \sim 10^{15} M_{Sun}/h$



Stacked weak lensing

- significant detection of weak lensing can be made only for massive clusters at relevant redshifts (z~0.2-0.5)
- stacked weak lensing technique allows weak lensing studies for less massive clusters (or galaxies) at higher redshifts
- it is important especially in the era of wide-field imaging surveys

Concept of stacked weak lensing



- combine shear measurements for different clusters to get constraints on average property

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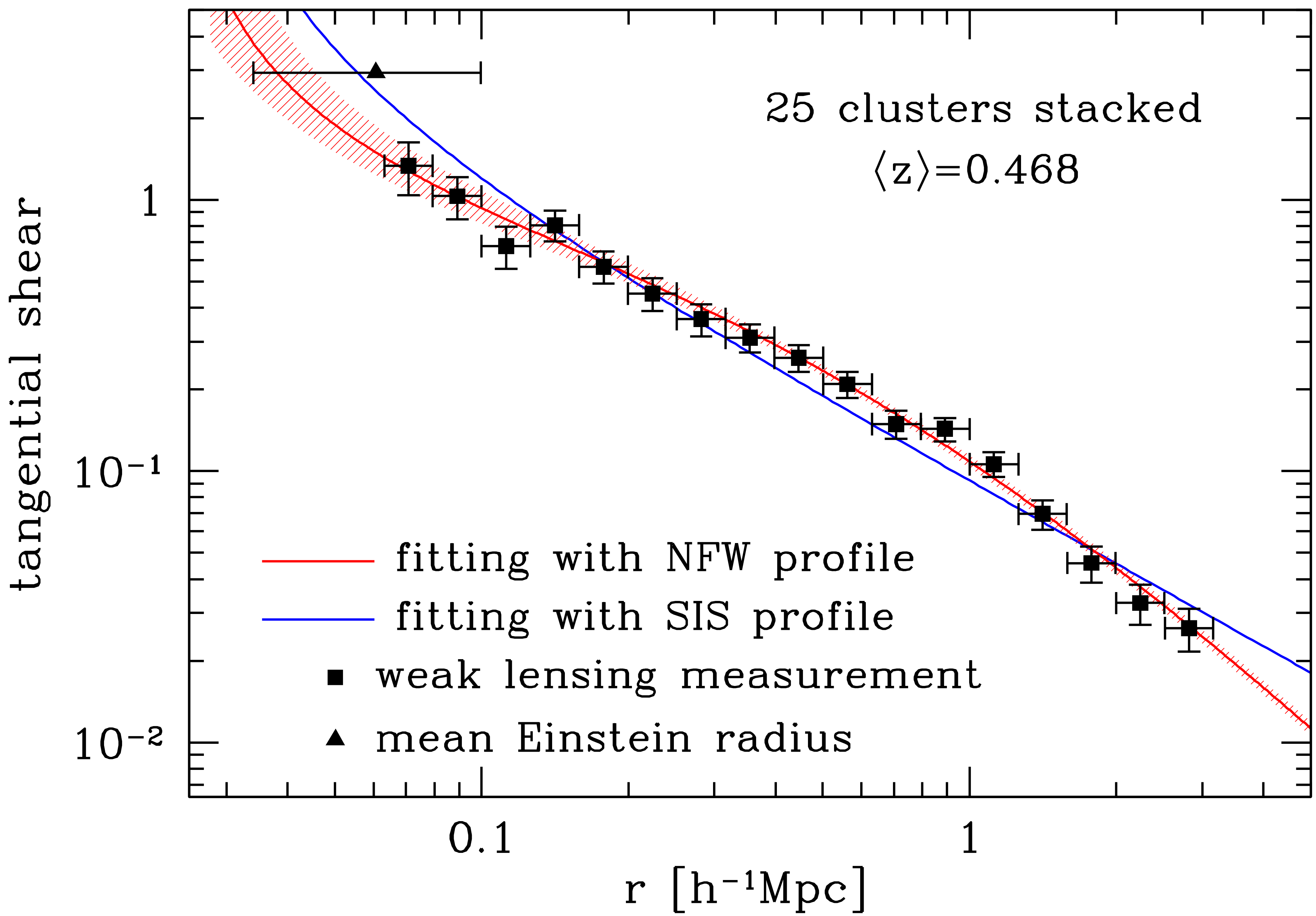
Power of stacked weak lensing (I)

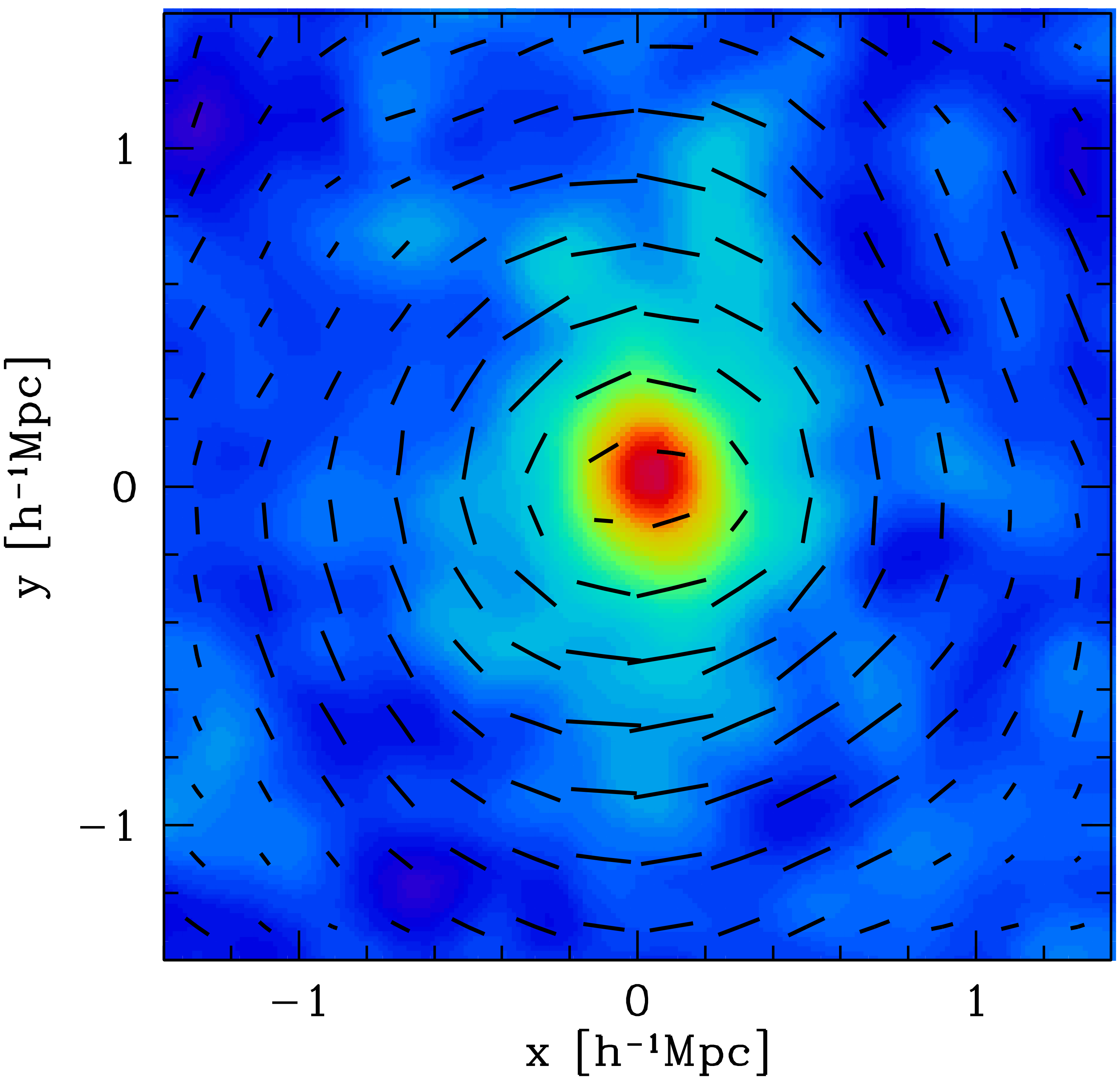


very precise shear profile!

profile very well fitted by NFW profile

Oguri et al. MNRAS, **420**, 3213 (2012)

Power of stacked weak lensing (II)

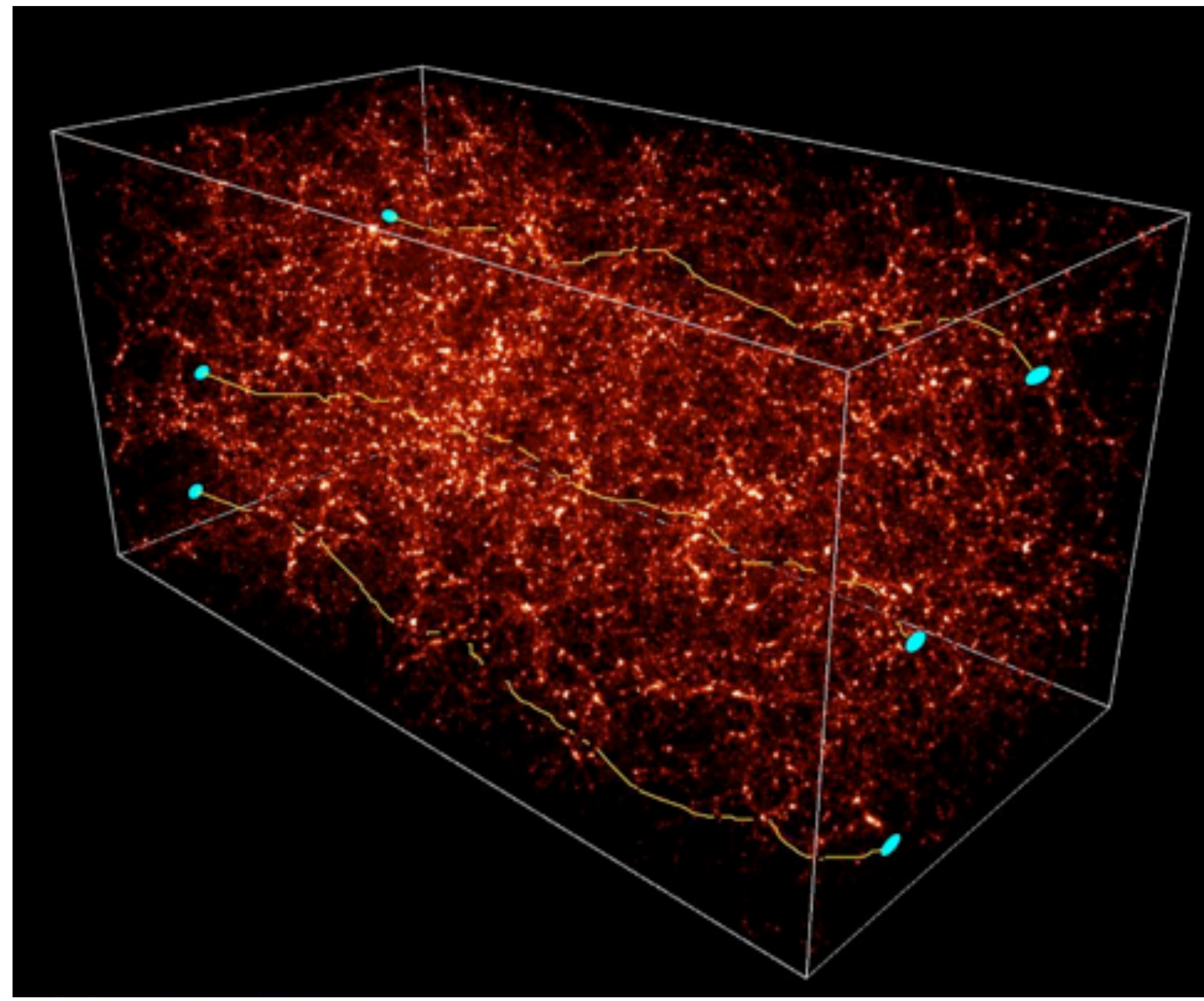


2D stacking of 25 clusters

distribution of dark matter in clusters is not spherical but highly elongated (axis ratio ~ 0.5), consistent with ΛCDM prediction

Weak lensing by large-scale structure

- direct mapping of cosmological dark matter distribution via weak lensing
- can be compared with simulations directly, powerful cosmological probe



Cosmological weak lensing (I)

- recall: convergence is written as

$$\kappa(\vec{\theta}) = \int d\chi W_{\mathrm{GL}}(\chi)\delta(\chi, \vec{\theta})$$

$$W_{\mathrm{GL}}(\chi) = \frac{3\Omega_M H_0^2}{2c^2} \frac{f_K(\chi_s - \chi) f_K(\chi)}{f_K(\chi_s)} a$$

- angular correlation function

$$w^{\kappa\kappa}(\theta) \equiv \langle \kappa(\vec{\theta}') \kappa(\vec{\theta}' + \vec{\theta}) \rangle$$

$$= \int d\chi W_{\mathrm{GL}}(\chi) \int d\chi' W_{\mathrm{GL}}(\chi') \langle \delta(\vec{x}) \delta(\vec{x}') \rangle$$

Cosmological weak lensing (II)

- work in Fourier space

$$\delta(\vec{x}) = \int \frac{d\vec{k}}{(2\pi)^3} \hat{\delta}(\vec{k}) e^{i\vec{k}\cdot\vec{x}}$$

$$\langle \hat{\delta}(\vec{k}) \hat{\delta}(\vec{k}') \rangle = (2\pi)^3 \delta(\vec{k} + \vec{k}') P(k)$$

P(k): matter power spectrum

- Rayleigh's formula

$$e^{i\vec{k}\cdot\vec{x}} = 4\pi \sum_{\ell,m} i^\ell j_\ell(k f_K(\chi)) Y_{\ell m}(\vec{\theta}) Y^*_{\ell m}(\vec{n}_k)$$

$j_\ell(x)$: spherical Bessel func.
$Y_{\ell m}(x)$: spherical harmonics

- orthogonal relation

$$\int d\Omega_k Y_{\ell m}(\vec{n}_k) Y^*_{\ell' m'}(\vec{n}_k) = \delta_{\ell\ell'} \delta_{mm'}$$

Cosmological weak lensing (III)

- addition theorem

$P_\ell(x)$: Legendre polynomials

$$P_\ell(\cos\theta) = \frac{4\pi}{2\ell+1}\sum_m Y_{\ell m}(\vec{\theta}')Y^*_{\ell m}(\vec{\theta}'+\vec{\theta})$$

- then angular correlation function becomes

$$w^{\kappa\kappa}(\theta) = \sum_\ell \frac{2\ell+1}{4\pi}C^{\kappa\kappa}(\ell)P_\ell(\cos\theta) \to \int \frac{\ell d\ell}{2\pi}C^{\kappa\kappa}(\ell)J_0(\ell\theta)$$

(small angle approx.)

$J_0(x)$: zeroth Bessel func.

$$C^{\kappa\kappa}(\ell) = \int d\chi W_{\rm GL}(\chi)\int d\chi' W_{\rm GL}(\chi')\frac{2}{\pi}\int k^2 dk P(k) j_\ell(kf_K(\chi))j_\ell(kf_K(\chi'))$$

$C^{\kappa\kappa}$: convergence power spectrum

Limber's approximation

- use the following equality

$$\frac{2}{\pi}\int k^2 dk P(k) j_\ell(k f_K(\chi)) j_\ell(k f_K(\chi')) = \frac{1}{f_K^2(\chi)}\delta(f_K(\chi) - f_K(\chi'))$$

- assuming that P(k) is slowly-varying with k

$$C^{\kappa\kappa}(\ell) = \int d\chi W_{\rm GL}^2(\chi)\frac{1}{f_K^2(\chi)}P(k = \ell/f_K(\chi))$$

convergence power spectrum

matter power spectrum

Connection to shear 2PCF

- shear is related to convergence as

$$\hat{\gamma}_1(\vec{\ell}) = \cos(2\phi_\ell)\hat{\kappa}(\vec{\ell}) \qquad \hat{\gamma}_2(\vec{\ell}) = \sin(2\phi_\ell)\hat{\kappa}(\vec{\ell})$$

- therefore two-point correlation function of shear can be described by $C^{\kappa\kappa}(\ell)$

$$\xi_+(\theta) \equiv w^{\gamma_+\gamma_+}(\theta) + w^{\gamma_\times\gamma_\times}(\theta) = \int \frac{\ell d\ell}{2\pi} C^{\kappa\kappa}(\ell) J_0(\ell\theta)$$

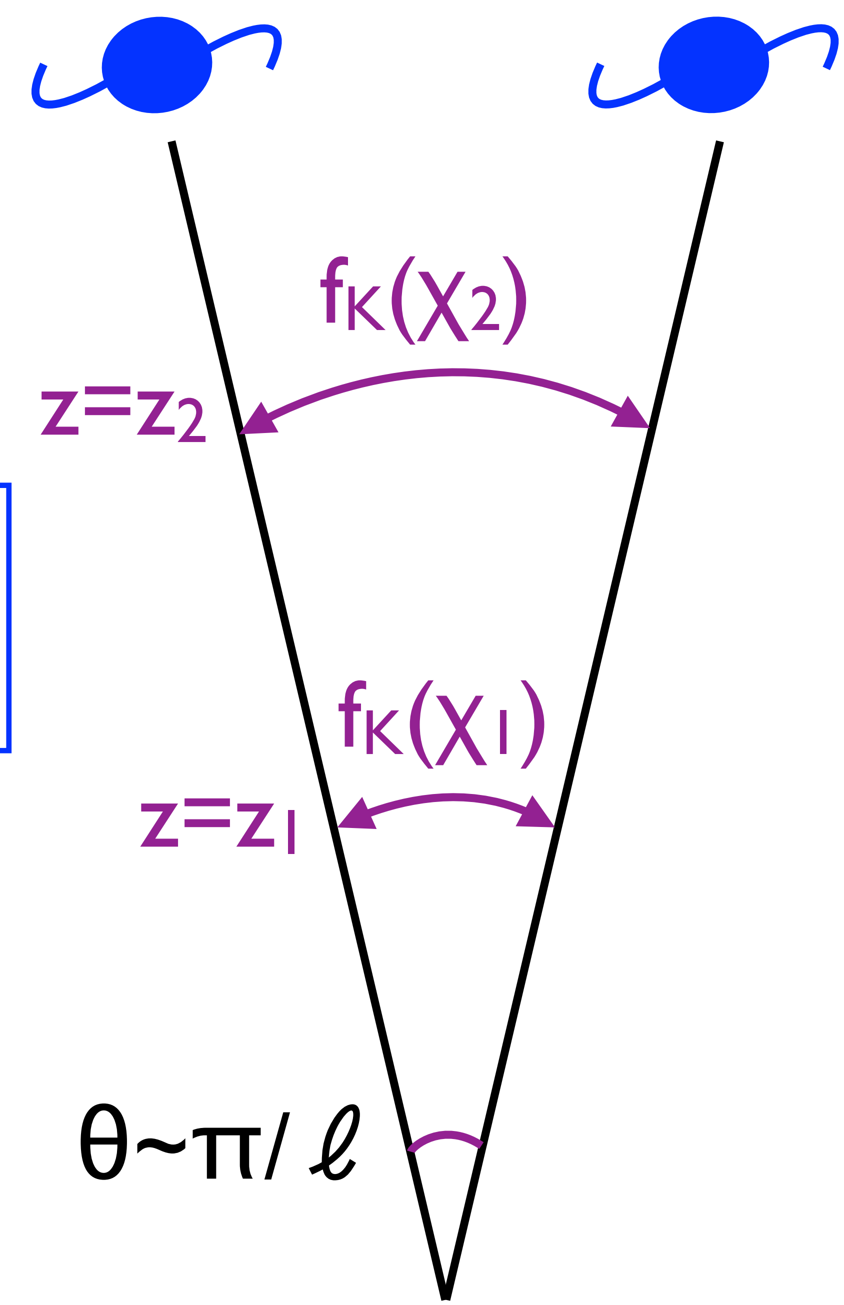
$$\xi_-(\theta) \equiv w^{\gamma_+\gamma_+}(\theta) - w^{\gamma_\times\gamma_\times}(\theta) = \int \frac{\ell d\ell}{2\pi} C^{\kappa\kappa}(\ell) J_4(\ell\theta)$$

Physical interpretation

- convergence power spectrum is integral of matter power spectrum P(k) along l.o.s.

$$C^{\kappa\kappa}(\ell) = \int d\chi W^2_{\rm GL}(\chi) \frac{1}{f^2_K(\chi)} P(k = \ell / f_K(\chi))$$

- however wavelength k varies with redshift, i.e., weak lensing mixes up different k-mode (therefore no 'BAO' seen)



Summary

- weak lensing measures reduced shear by averaging many galaxies' shapes
- fit measured shear with model predictions, or direct inversion technique to reconstruct a mass (convergence κ) map
- signals enhanced by stacking many lenses
- weak lensing correlation function (power spectrum) probe matter power spectrum

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