Applications of gravitational lensing in astrophysics and cosmology

2. Strong lensing analysis

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1. Introduction & basics of gravitational lensing
2. $\boxed{\text{Strong lensing analysis}}$
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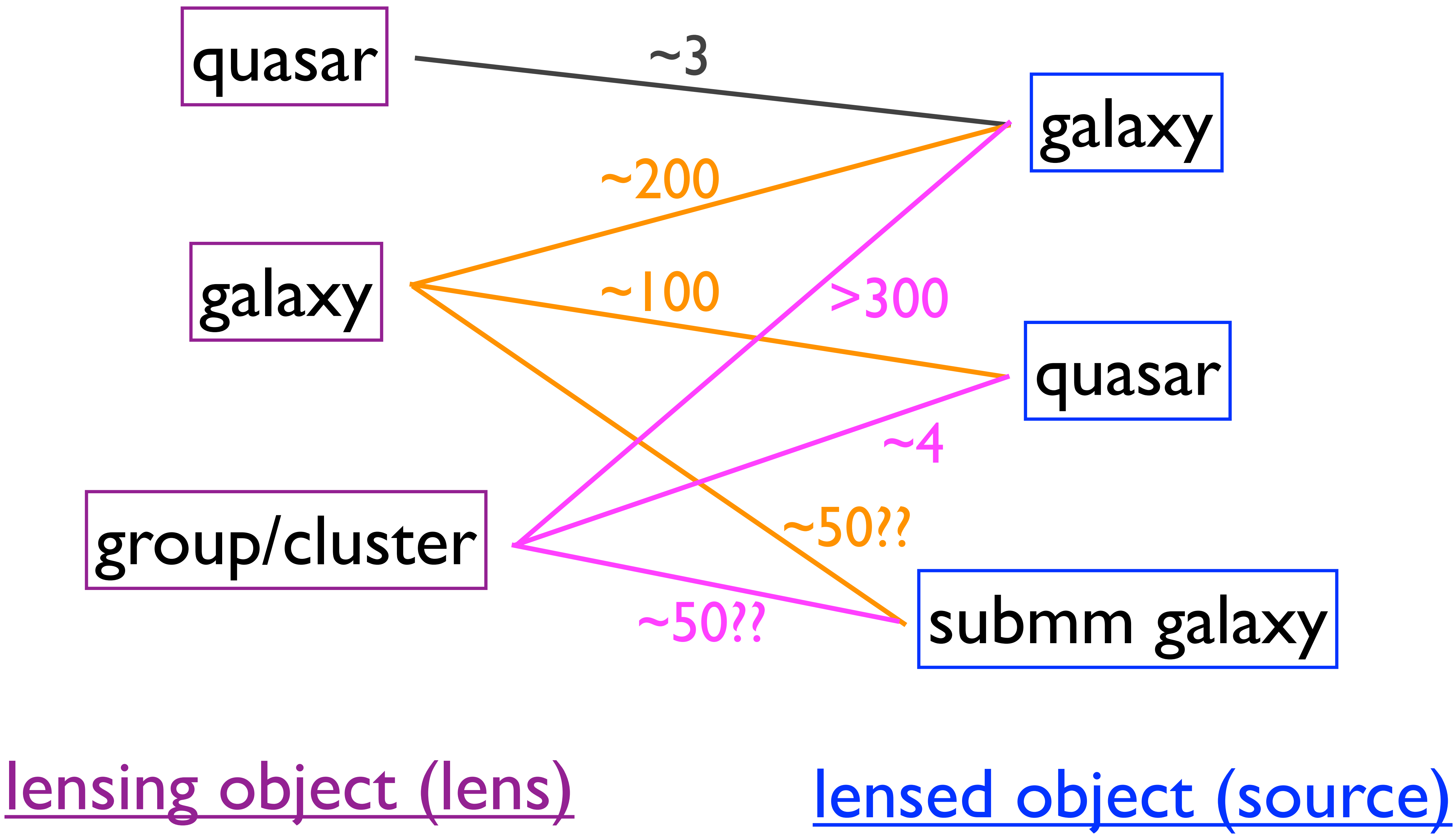
4. Cosmological applications

Strong vs weak lensing

- strong lensing
  - observed for individual sources
  - $\kappa \gtrsim 1$ ($\Sigma \gtrsim \Sigma_{cr}$), near critical curves/caustics
  - multiple images, high elongation/magnification
- weak lensing
  - observed for ensemble of sources
  - $\kappa \ll 1$ ($\Sigma \ll \Sigma_{cr}$), far from critical curves/caustics
  - no multiple image, tiny elongation/magnification

(partly) based on Master Lens Database

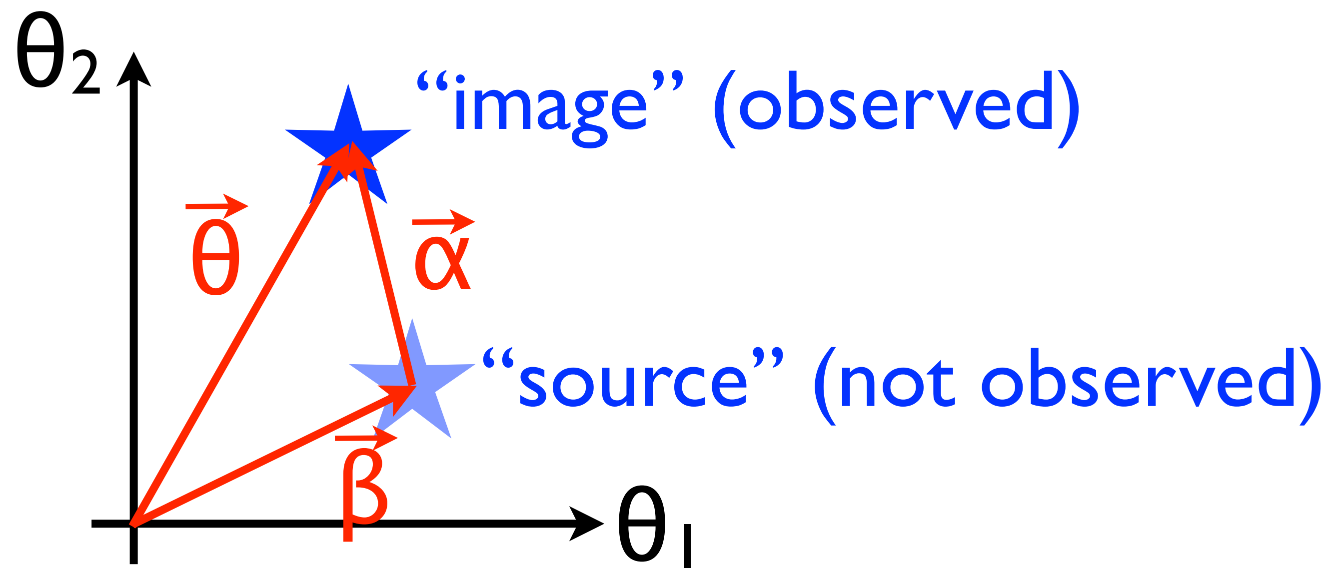
Strong lens kinds



Challenge in strong lensing analysis

- lens equation is 'mapping' between β and θ

$$\boxed{\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})}$$



- in many cases we want to know $\vec{\theta}$ from $\vec{\beta}$, but it is in general very difficult because
  – lens equation is non-linear in $\vec{\theta}$
  – solution is not unique (multiple images!)

Strong lensing analysis

- circular symmetric lenses
- more realistic models
- numerical approach
- modeling strong lens systems

Circular symmetric lenses (I)

- simple yet useful

$$\kappa(\vec{\theta}) = \kappa(\theta) \qquad |\vec{\theta}| = \theta$$

then lens potential ψ becomes

$$\psi(\vec{\theta}) = \frac{1}{\pi}\int_0^{\infty} d\theta'\theta'\kappa(\theta')\int_0^{2\pi} d\phi \ln\left|\vec{\theta} - \vec{\theta}'\right|$$

$$= 2\int_0^{\theta} d\theta'\theta'\kappa(\theta') \ln\left(\frac{\theta}{\theta'}\right)$$

$$\rightarrow \psi(\vec{\theta}) = \psi(\theta)$$

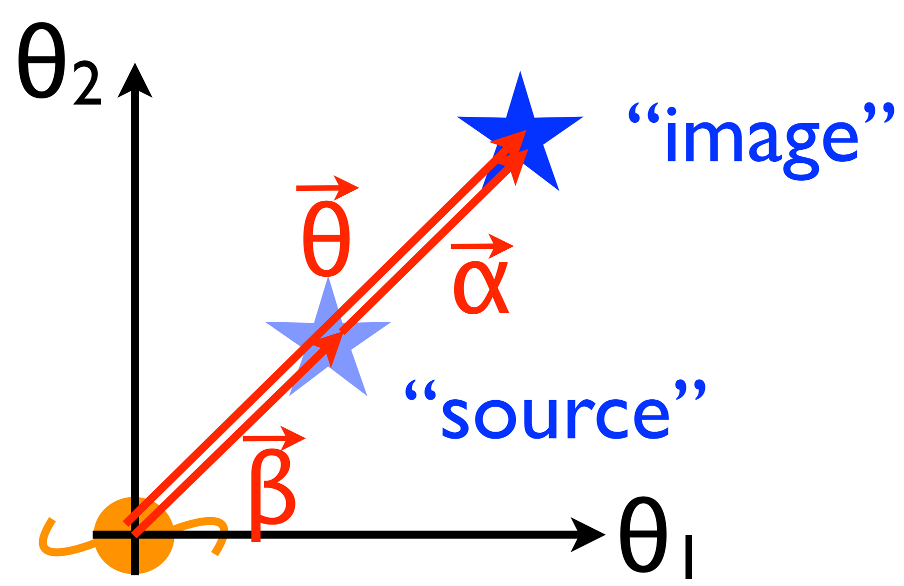
Circular symmetric lenses (II)

- deflection angle

$$\vec{\alpha}(\vec{\theta}) = \vec{\nabla}\psi(\theta) = \underbrace{\left[\frac{2}{\theta^2}\int_0^{\theta} d\theta'\theta'\kappa(\theta')\right]}_{=\bar{\kappa}(<\theta)}\vec{\theta}$$

$\rightarrow \vec{\alpha}(\vec{\theta}) \parallel \vec{\theta}$

$$\alpha(\theta) = \theta\,\bar{\kappa}(<\theta)$$

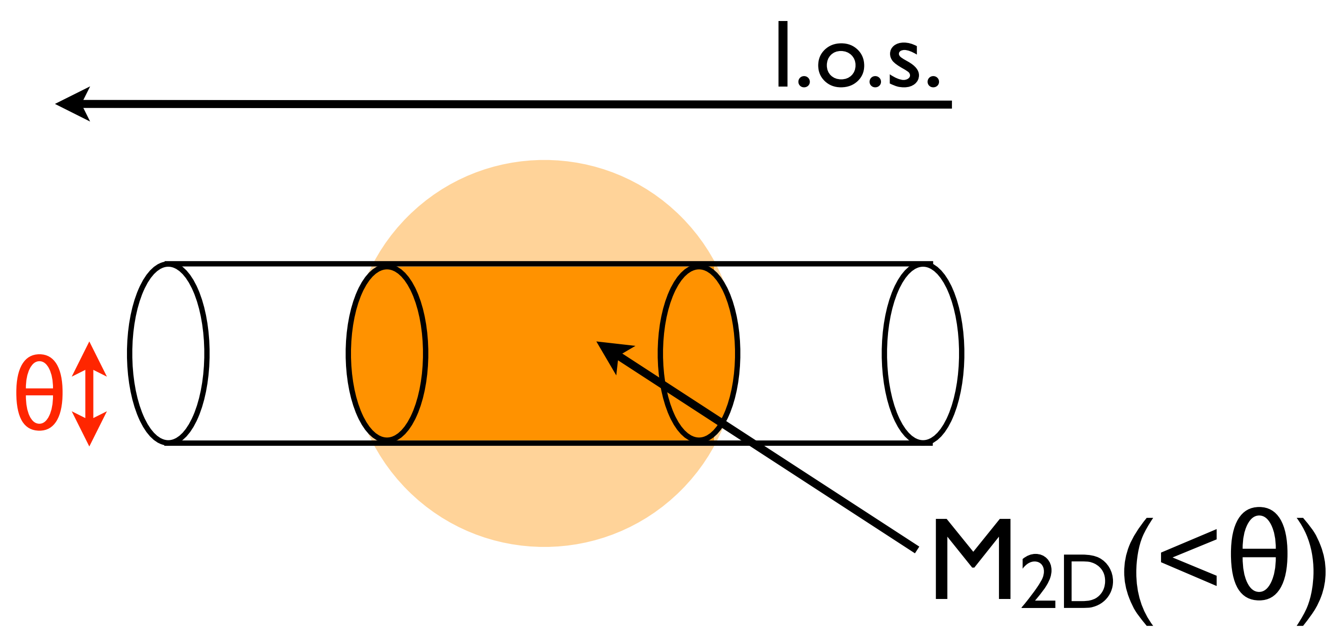


Circular symmetric lenses (III)

- therefore, lens equation reduces to 1D eq.

$$\boxed{\beta = \theta - \alpha(\theta) = \left[1 - \bar{\kappa}(<\theta)\right]\theta}$$

note: $\bar{\kappa}(<\theta) = \dfrac{M_{2D}(<\theta)}{\pi\theta^2 D_A^2(z_l)\Sigma_{\rm cr}}$



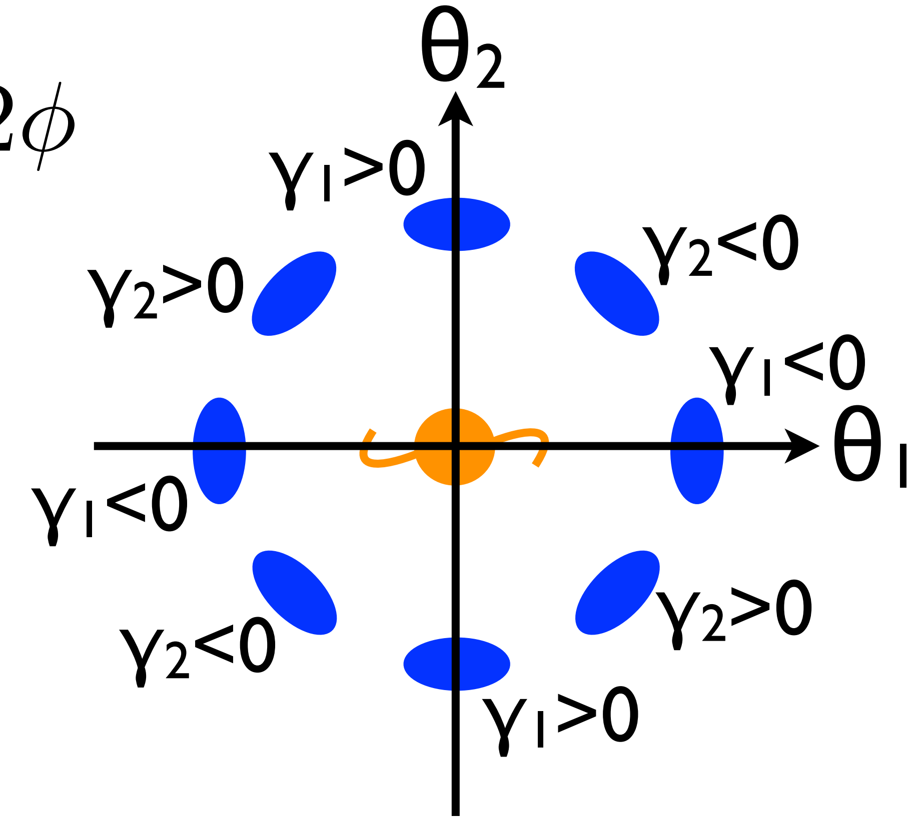
Circular symmetric lenses (IV)

- shear   [polar coords (θ_1, θ_2)=($\theta\cos\phi$, $\theta\sin\phi$)]

using the relation: $\bar{\kappa}'(<\theta) = -\frac{2}{\theta}\left[\bar{\kappa}(<\theta) - \kappa(\theta)\right]$

$$\gamma_1 = \frac{1}{2}\left(\frac{\partial\alpha_1}{\partial\theta_1} - \frac{\partial\alpha_2}{\partial\theta_2}\right) = -\left[\bar{\kappa}(<\theta) - \kappa(\theta)\right]\cos 2\phi$$

$$\gamma_2 = \frac{\partial\alpha_1}{\partial\theta_2} = -\left[\bar{\kappa}(<\theta) - \kappa(\theta)\right]\sin 2\phi$$



lens object centered at $\theta \approx 0$

$\rightarrow \bar{\kappa}(<\theta) - \kappa(\theta) > 0$

Circular symmetric lenses (V)

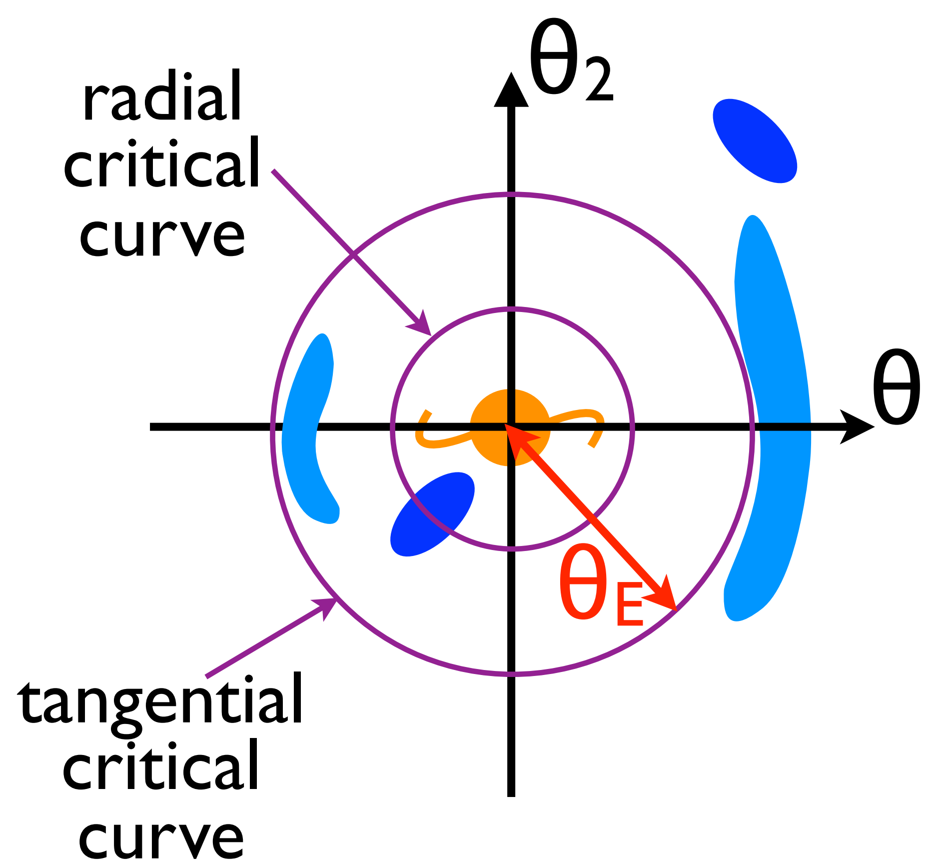
- critical curves

$$\det A = (1-\kappa)^2 - |\gamma|^2 = \underbrace{[1-\bar{\kappa}(<\theta)]}_{\text{tangential critical curve}}\underbrace{[1+\bar{\kappa}(<\theta)-2\kappa(\theta)]}_{\text{radial critical curve}}$$

tangential critical curve is a solution for β=0

$$\bar{\kappa}(<\theta_{\rm E}) = 1$$

θ_E: Einstein radius

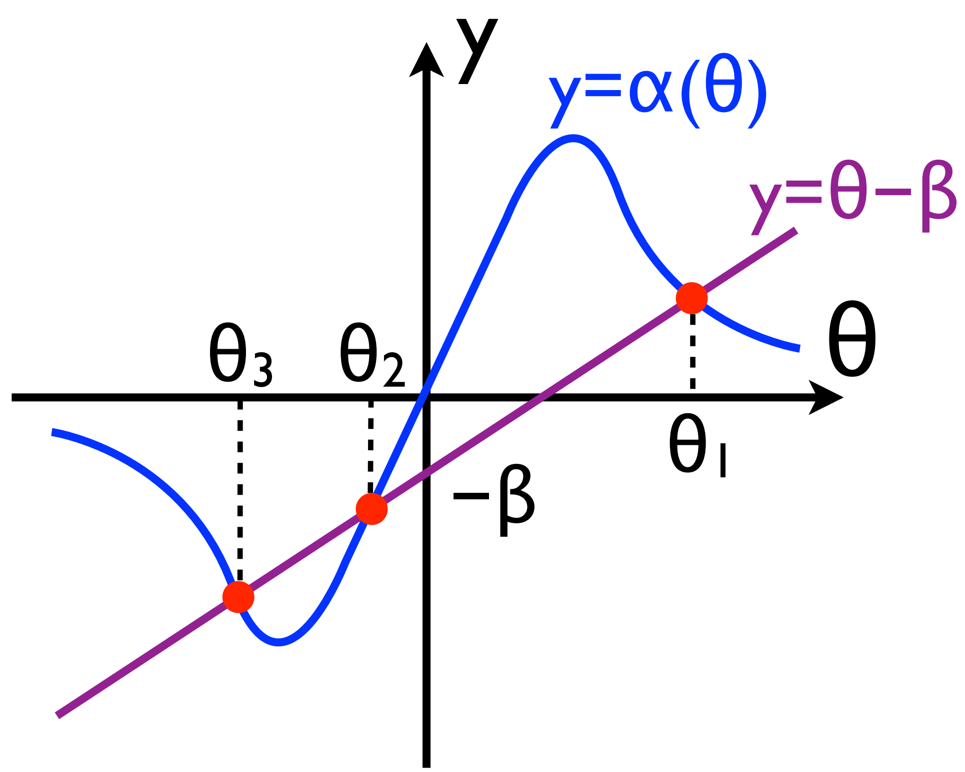


Solutions of lens equation

- lens equation is 1D equation
- ‘diagrammatic’ approach is useful to understand how multiple solutions appear

$$\beta = \theta - \alpha(\theta)$$

$$\Leftrightarrow \begin{cases} y = \alpha(\theta) \\ y = \theta - \beta \end{cases}$$

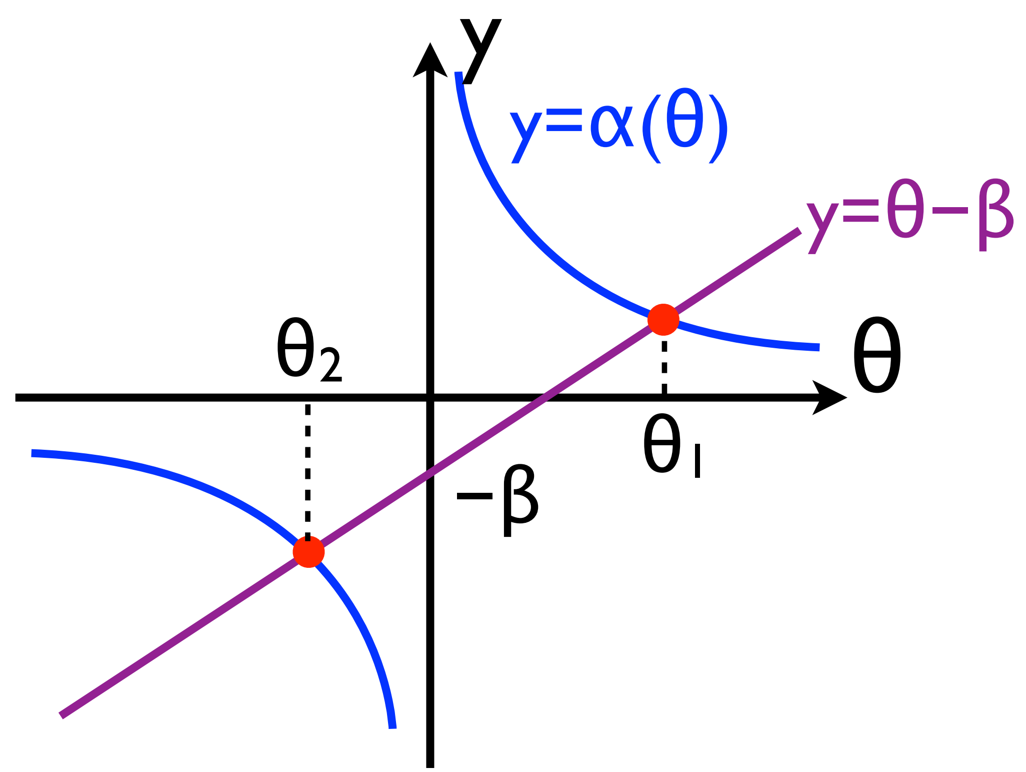


Example 1: point mass

- model for stars, compact galaxies, ...

$$\bar{\kappa}(<\theta) = \underbrace{\frac{M}{\pi D_A^2(z_l)\Sigma_{\rm cr}}}_{\equiv\,\theta_{\rm E}^2}\frac{1}{\theta^2} = \frac{\theta_{\rm E}^2}{\theta^2}$$

$$\alpha(\theta) = \theta\,\bar{\kappa}(<\theta) \propto \frac{1}{\theta}$$



$\boxed{\textit{always}\text{ two images}}$

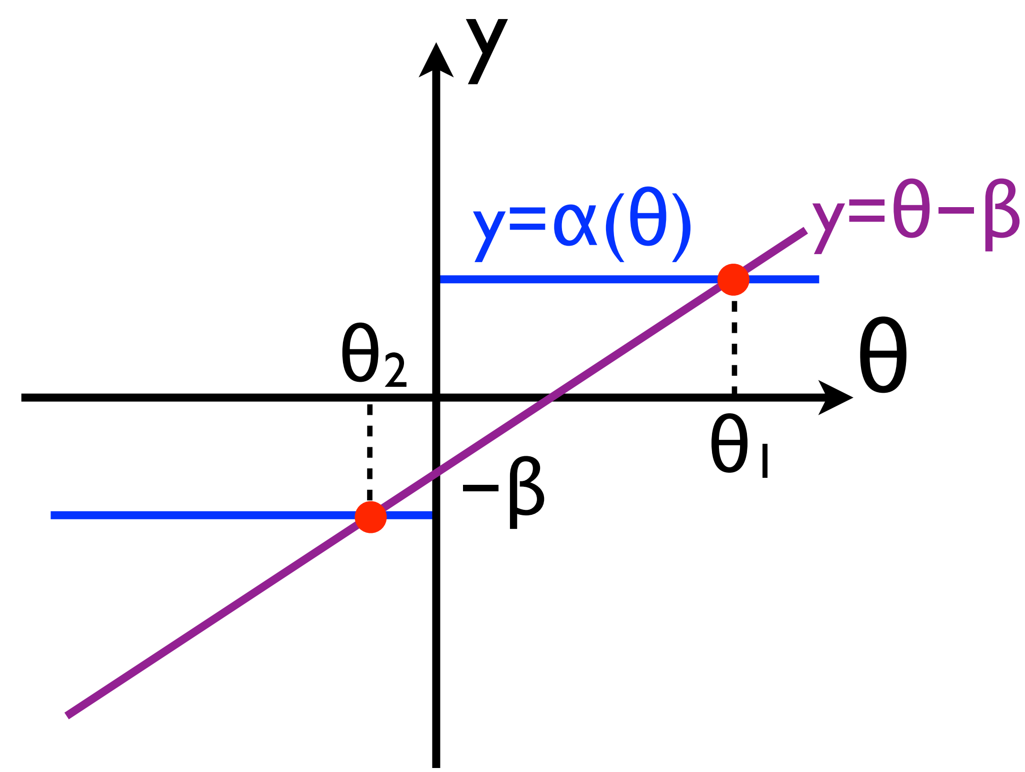
Example 2: singular isothermal sphere

- standard lens model for galaxies

$$\rho(r) = \frac{\sigma^2}{2\pi G r^2} \qquad \Sigma(x) = \frac{\sigma^2}{2\pi G}\int_{-\infty}^{\infty}\frac{dz}{x^2+z^2} = \frac{\sigma^2}{2 G D_A(z_l)\theta}$$

$$\kappa(\theta) = 2\pi\left(\frac{\sigma}{c}\right)^2 \frac{D_A(z_l, z_s)}{D_A(z_s)}\frac{1}{\theta}$$

$$\bar{\kappa}(<\theta) = \underbrace{4\pi\left(\frac{\sigma}{c}\right)^2 \frac{D_A(z_l, z_s)}{D_A(z_s)}}_{\equiv\, \theta_{\rm E}}\frac{1}{\theta} = \frac{\theta_{\rm E}}{\theta}$$



two images when $|\beta|<\theta_E$
one image when $|\beta|>\theta_E$

Example 3: NFW profile

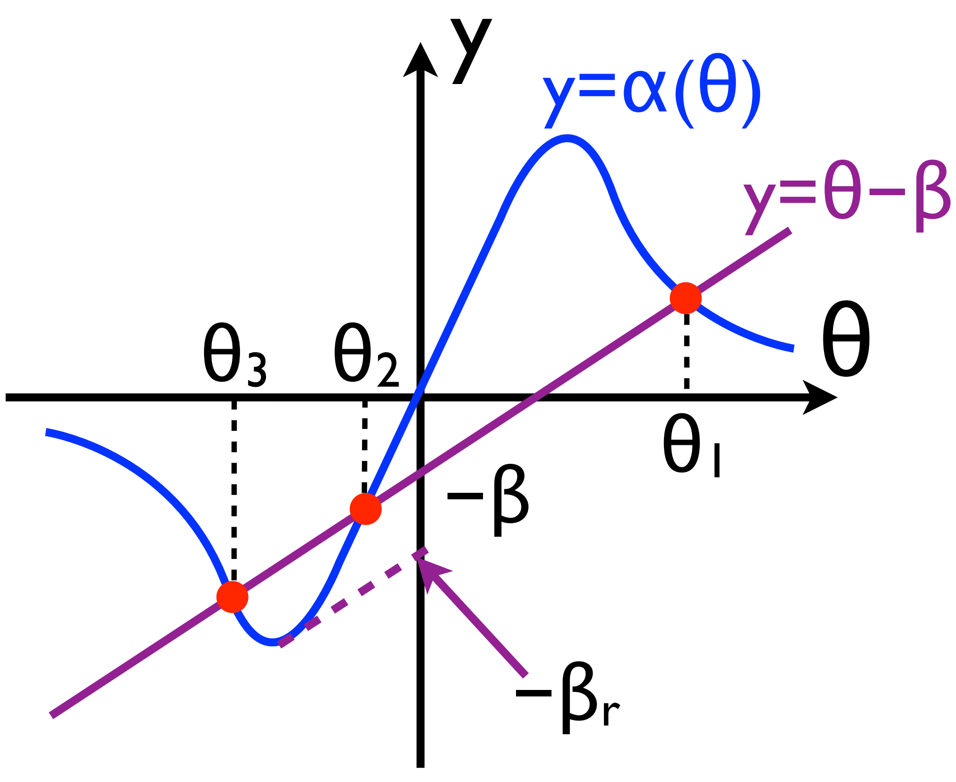
- standard lens model for dark matter halos

$$\rho(r) = \frac{\rho_s}{(r/r_s)(1+r/r_s)^2}$$

$$\bar{\kappa}(<\theta) \propto \begin{cases} \ln(\theta_s/\theta) & (\theta \ll \theta_s) \\ \theta^{-2} & (\theta \gg \theta_s) \end{cases}$$

$$\alpha(\theta) \propto \begin{cases} \theta\ln(\theta_s/\theta) & (\theta \ll \theta_s) \\ \theta^{-1} & (\theta \gg \theta_s) \end{cases}$$

three images when $|\beta|<\beta_r$
one image when $|\beta|>\beta_r$



More realistic models (I)

- elliptical lens $$\theta \to u \equiv \sqrt{\frac{\theta_1^2}{1-e} + (1-e)\theta_2^2}$$

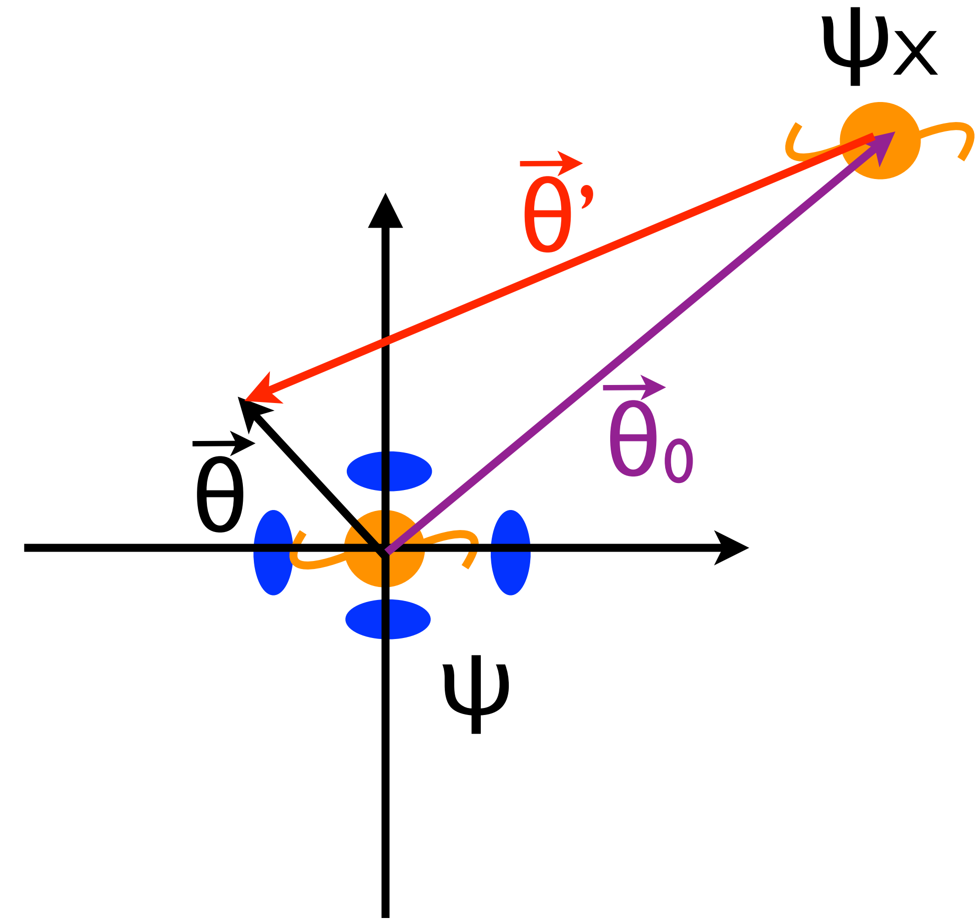
two approaches:

1. elliptical density $\kappa(u)$
   $\kappa(u) \to \psi(\vec{\theta}), \vec{\alpha}(\vec{\theta})$, ... through 1D integral
   computationally more expensive
2. elliptical potential $\psi(u)$
   can use circular sym. result, much easier,
   but can cause unphysical mass distributions
   ('dumbbell'-like κ map, negative κ, ...)

More realistic models (II)

- external perturbation

  nearby object (X) also contributes to the lens potential



$$\psi_{\mathrm{X}}(\vec{\theta}') = \psi_{\mathrm{X}}(\vec{\theta} - \vec{\theta}_0)$$

$$\approx \underbrace{\cancel{\psi_{\mathrm{X}}(-\vec{\theta}_0)}}_{\text{constant}} + \underbrace{\vec{\theta} \cdot \cancel{\left.\frac{\partial \psi_{\mathrm{X}}}{\partial \vec{\theta}}\right|_{-\vec{\theta}_0}}}_{\text{uniform } \vec{\alpha}} + \frac{1}{2}\vec{\theta} \cdot \underbrace{H\left[\psi_{\mathrm{X}}(-\vec{\theta}_0)\right]}_{\text{Hessian matrix}} \cdot \vec{\theta} + \cdots$$

$$H\left[\psi(\theta)\right] = \begin{pmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{pmatrix} = \begin{pmatrix} \kappa + \gamma_1 & \gamma_2 \\ \gamma_2 & \kappa - \gamma_1 \end{pmatrix}$$

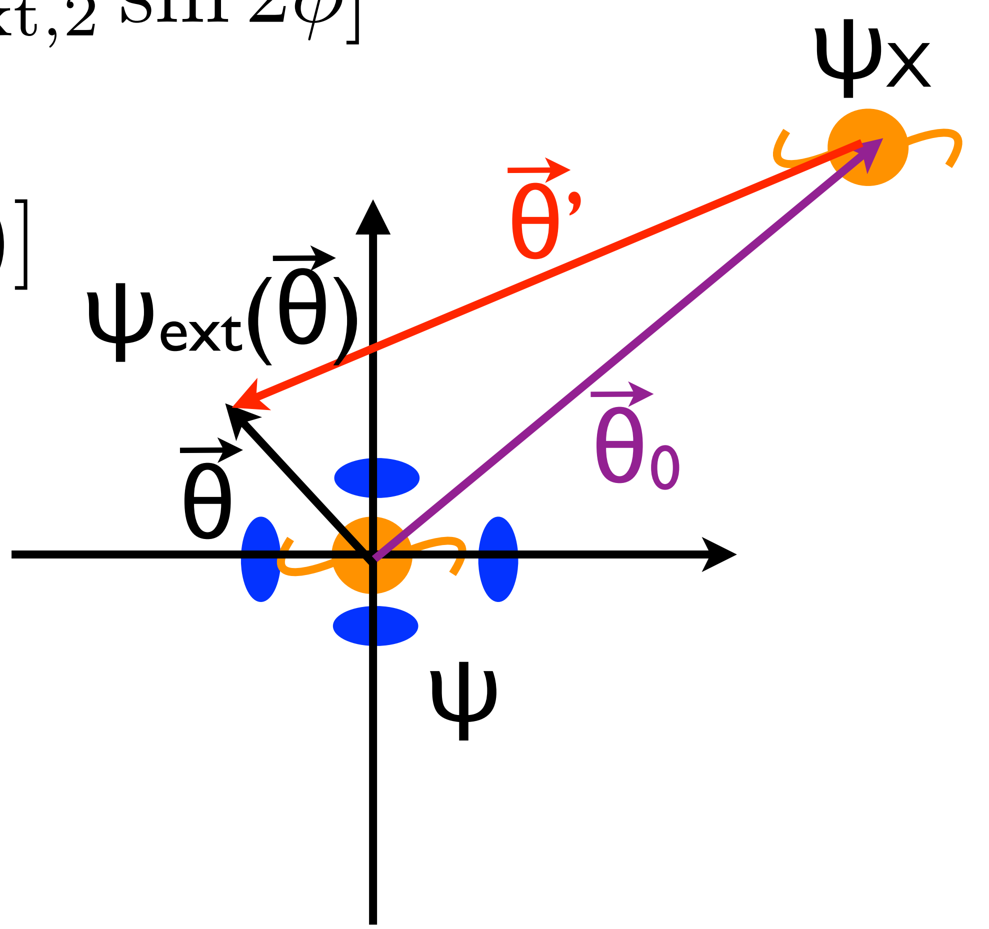
More realistic models (III)

- therefore, the effect of X on the main lens potential at $\vec{\theta}$, $\psi_{\rm ext}(\vec{\theta})=\psi_X(\vec{\theta}')$, becomes

[again, polar coords $(\theta_1, \theta_2)=(\theta\cos\phi, \theta\sin\phi)$]

$$\psi_{\rm ext}(\vec{\theta}) \approx \frac{\theta^2}{2}\left[\kappa_{\rm ext}+\gamma_{\rm ext,1}\cos 2\phi+\gamma_{\rm ext,2}\sin 2\phi\right]$$

$$\approx \frac{\theta^2}{2}\left[\kappa_{\rm ext}+\gamma_{\rm ext}\cos 2(\phi-\phi_0)\right]$$



(ϕ_0: polar angle corresponds to the direction to perturber)

Numerical approach

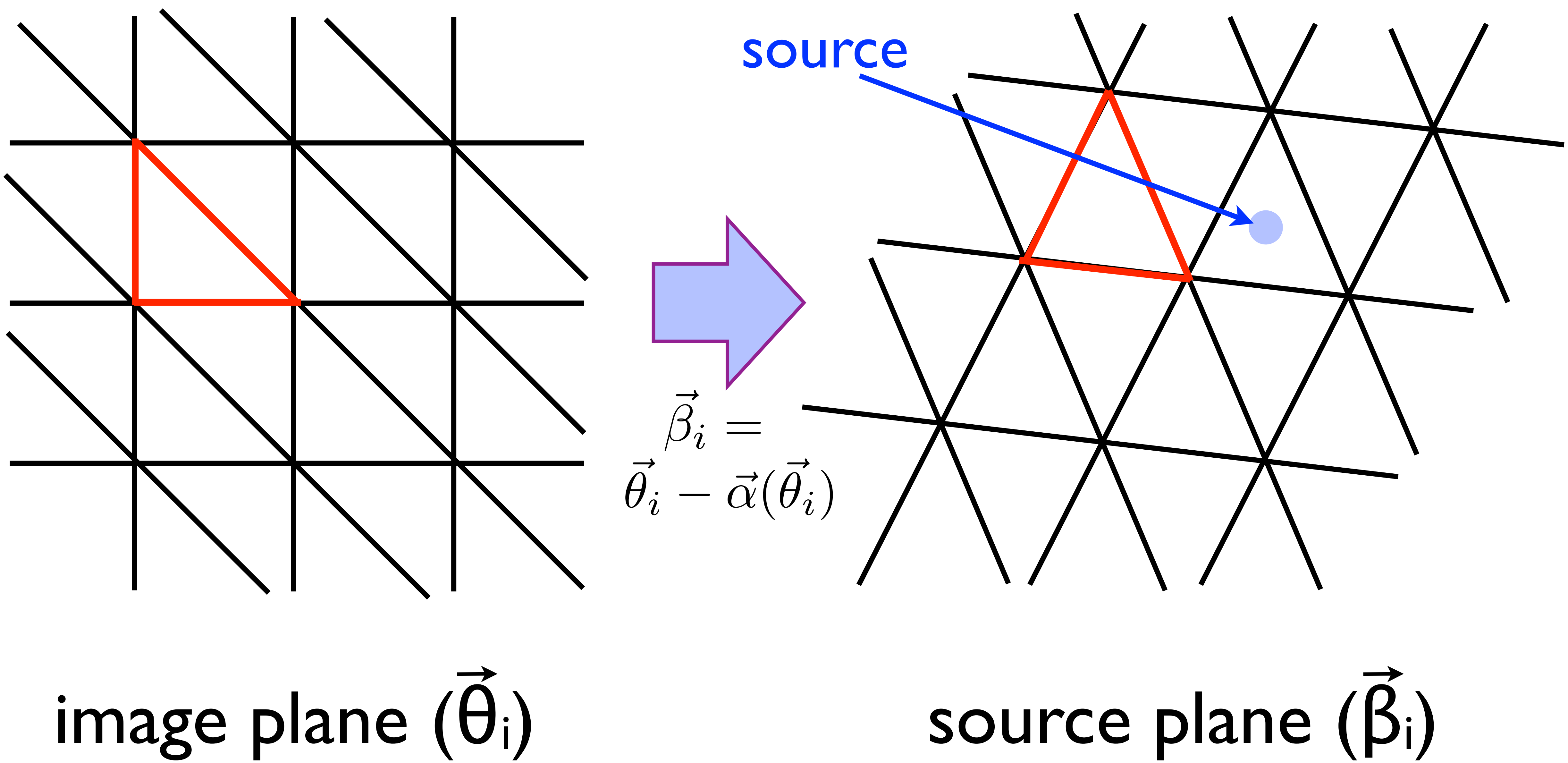
- recall: solving lens equation is hard in general

$$\boxed{\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})}$$

($\vec{\beta} \rightarrow \vec{\theta}$ is non-linear, multiple solutions allowed)

- numerical techniques to solve lens equation necessary

Numerical root finding



Numerical root finding

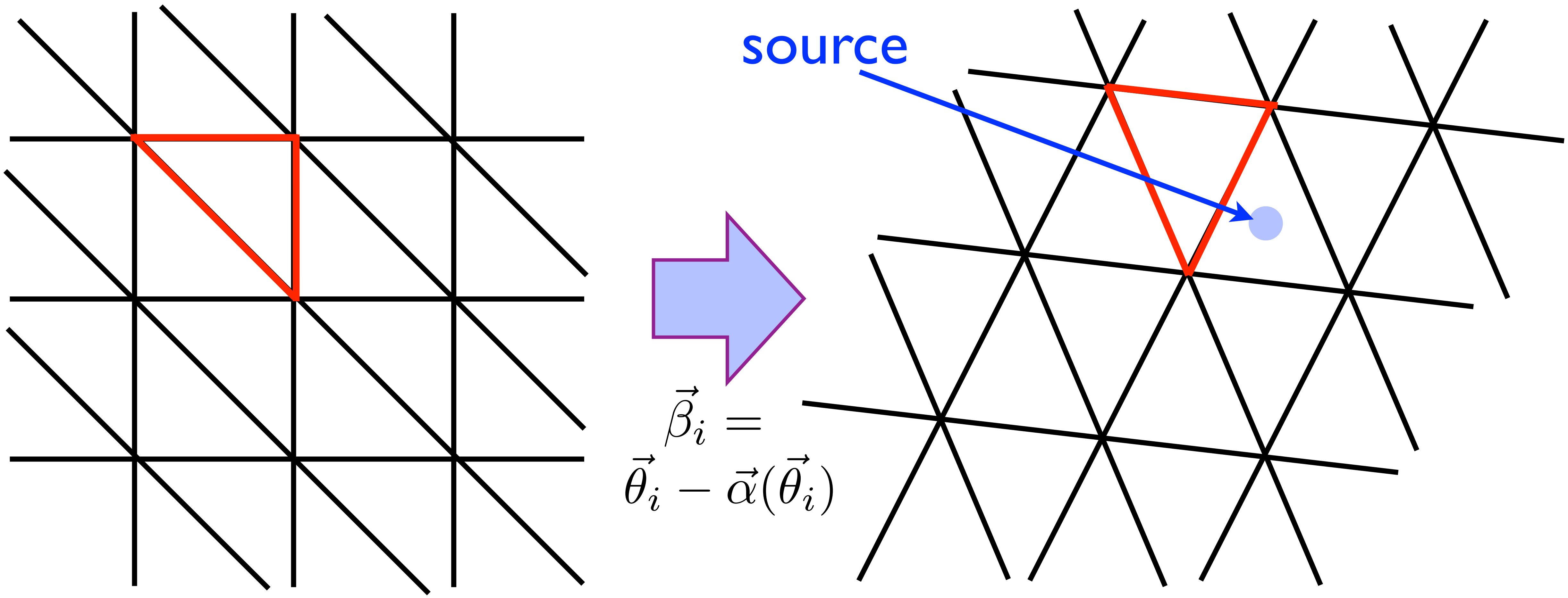
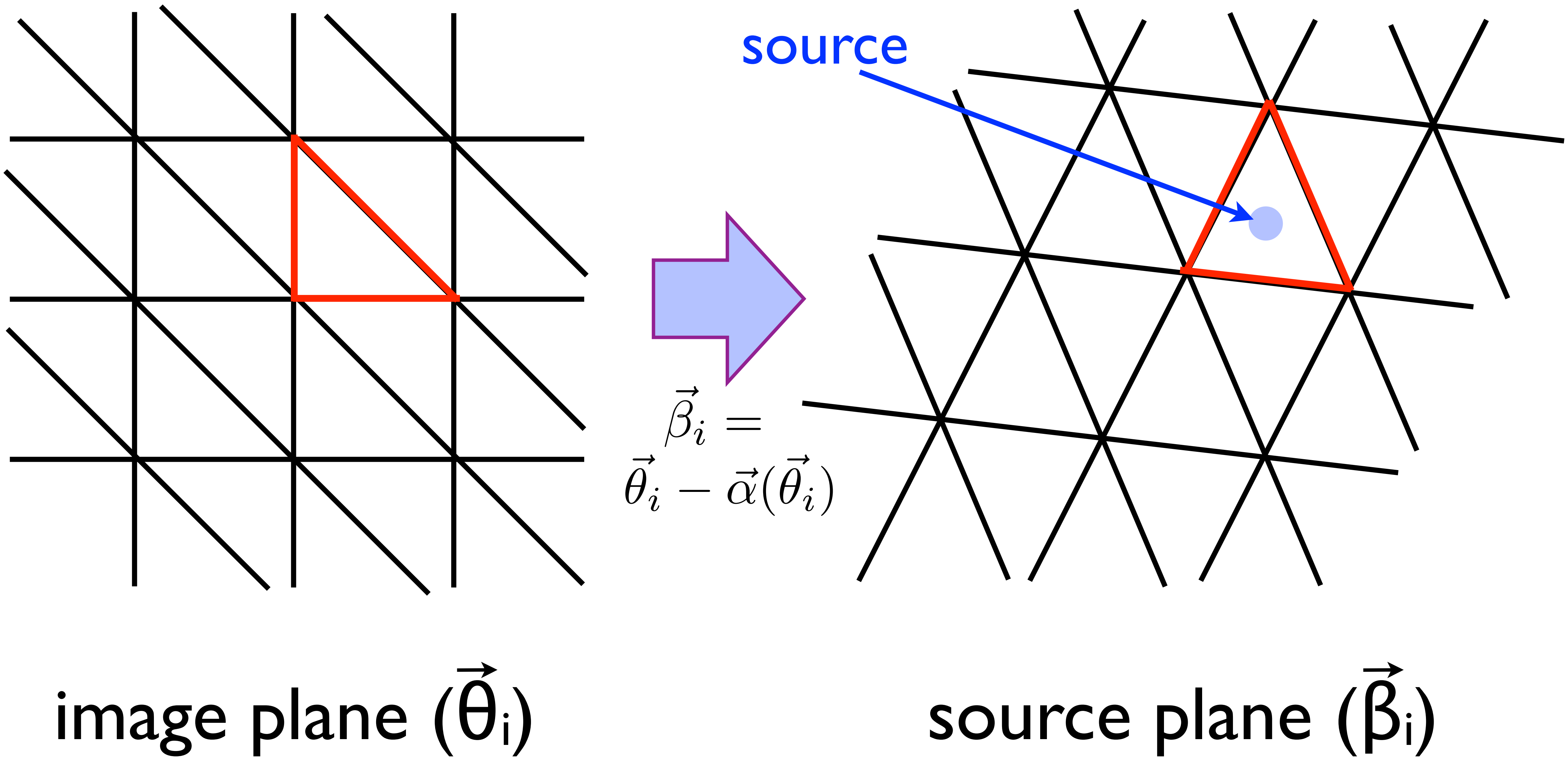


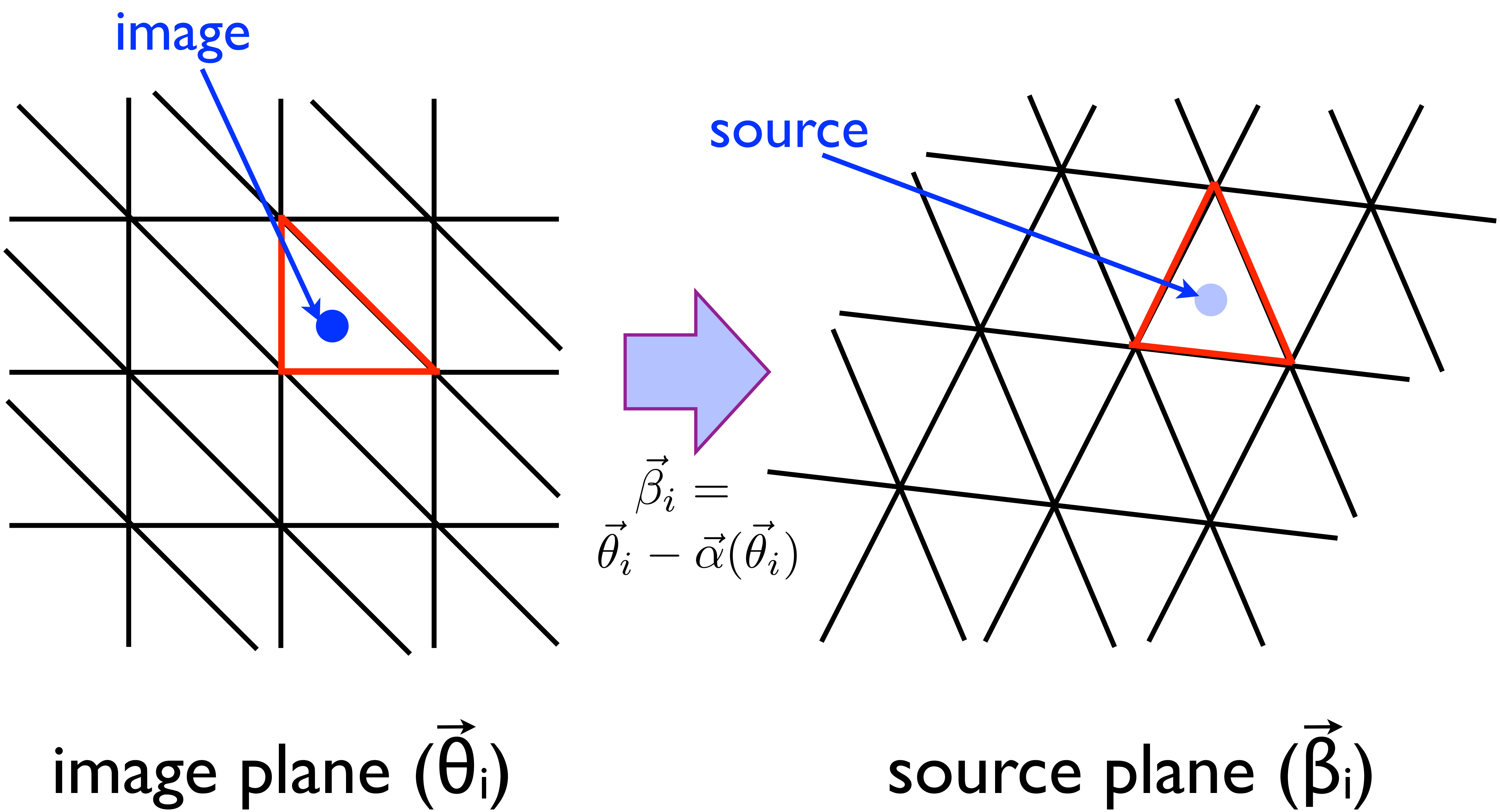
image plane ($\vec{\theta}_i$)

source plane ($\vec{\beta}_i$)

Numerical root finding



Numerical root finding



Resolution issue

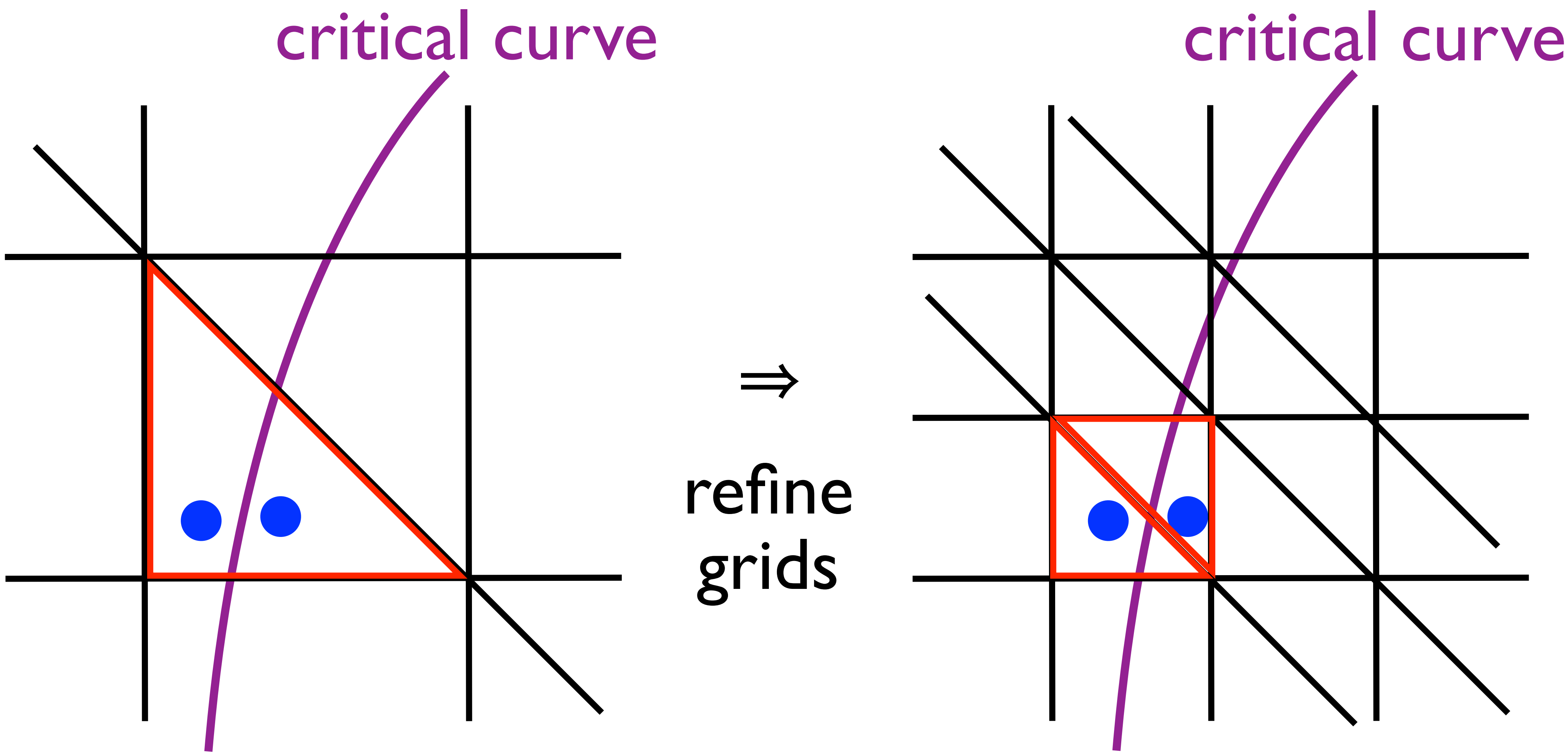
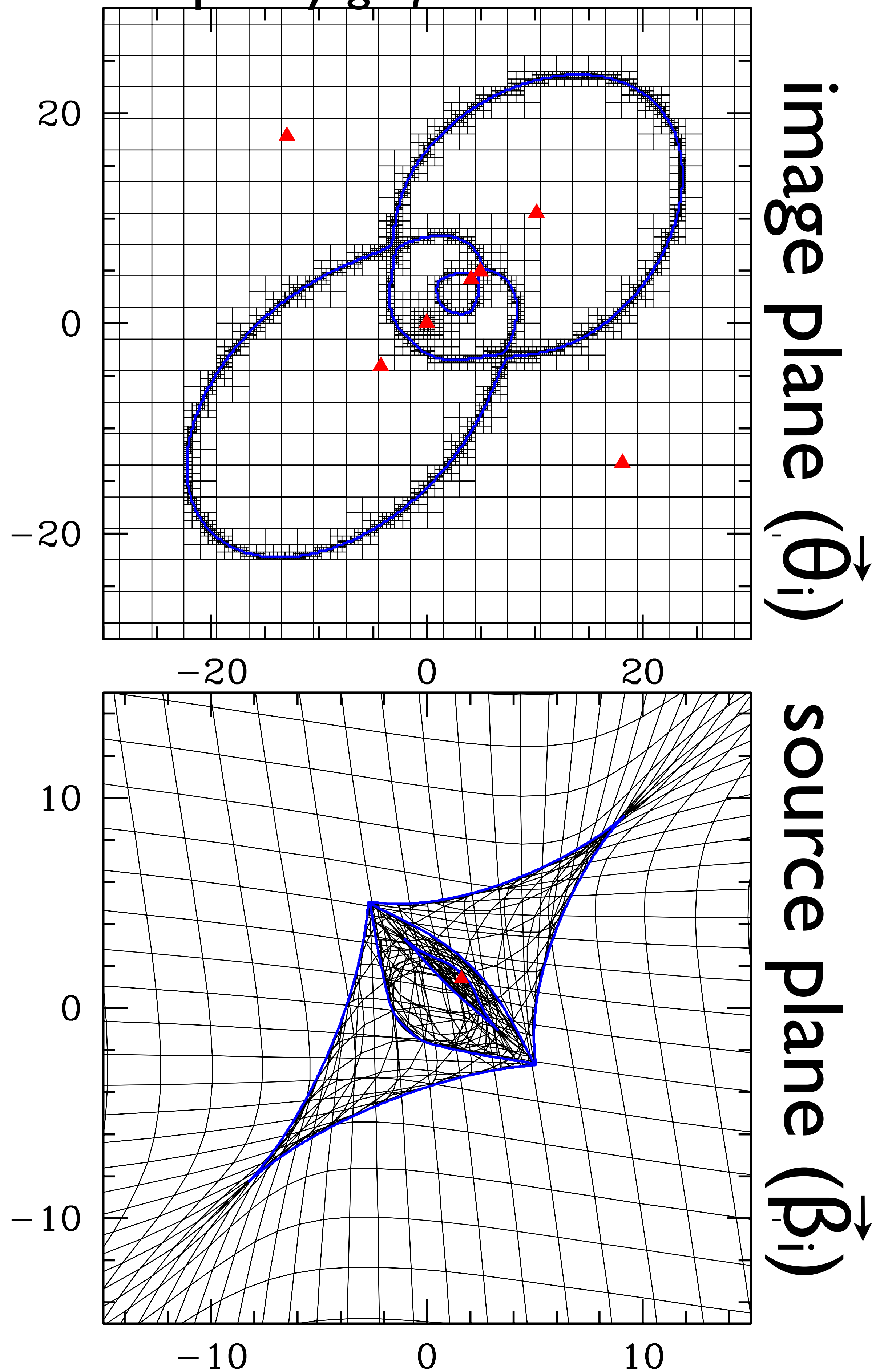


image plane ($\vec{\theta}_i$) image plane ($\vec{\theta}_i$)

fail to resolve multiple images

multiple images resolved

example by *glafic*



Practical cases

- very high grid resolution needed only near critical curves
- adaptive grid for efficient lens equation solving
- left example successfully identifies 7 lensed images of a single source

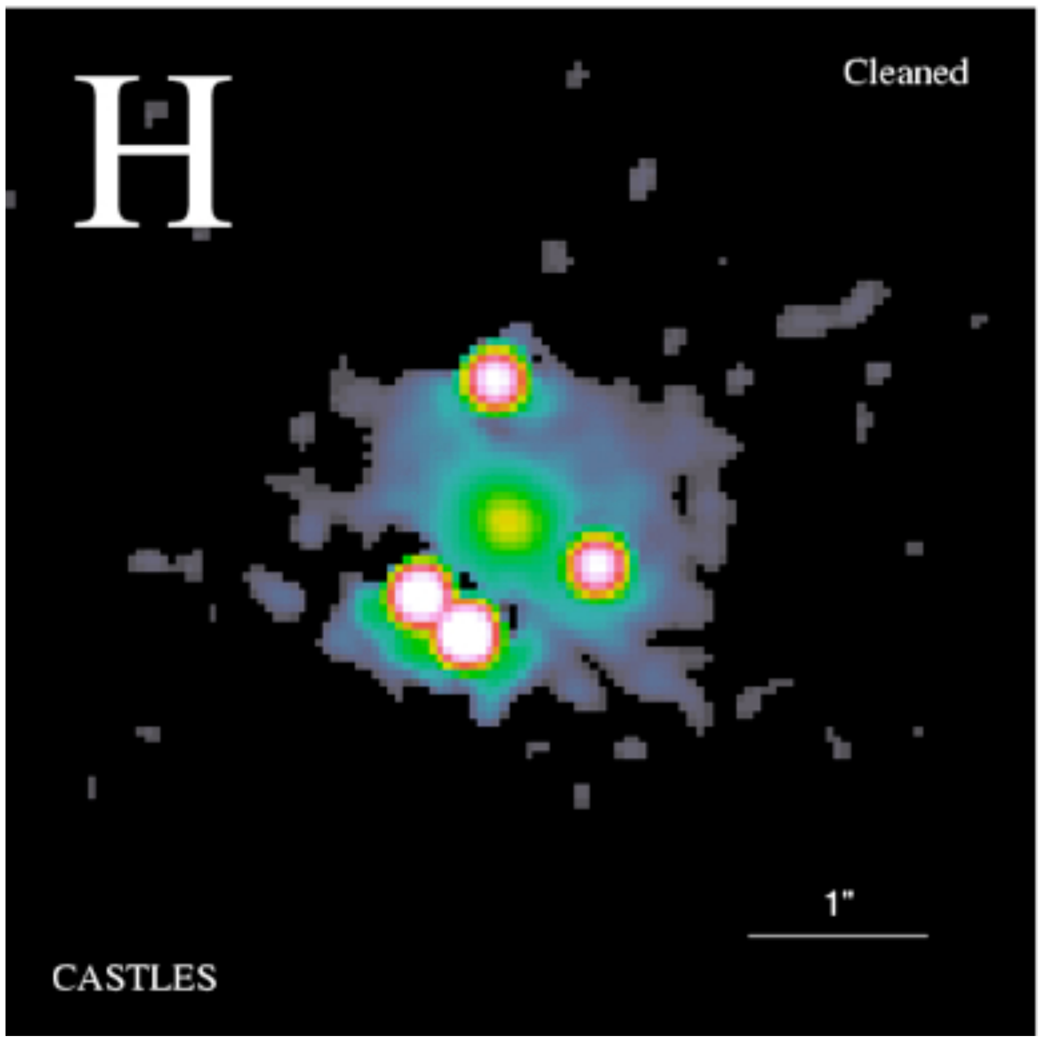
Public lens softwares

- public softwares that implement adaptive grid:
  - *glafic* (M. Oguri)
    http://www.slac.stanford.edu/~oguri/glafic/
  - *GRAVLENS* (C. R. Keeton)
    http://redfive.physics.rutgers.edu/~keeton/gravlens/
  - *LENSTOOL* (E. Jullo, J.-P. Kneib, et al.)
    http://lamwws.oamp.fr/lenstool/

    .....
- see also recent review of public softwares by Lefor et al. (arXiv:1206.4382)

Modeling strong lens systems (I)

- example: WFI2626-4536 (Morgan et al. 2004)



4 image system
source quasar at z=2.23
lensing galaxy at z~0.4

(HST image from CASTLES website)

Modeling strong lens systems (II)

- assume Singular Isothermal Ellipsoid (SIE) plus external shear
- model parameters = 9 (mass, SIE centroid, e, PA_e, γ_{ext}, PA_γ, $\vec{\beta}$)
- observational constraints = 13 (image position × 4, galaxy position, flux ratios × 3)
- degree of freedom = 13 − 9 = 4

Modeling strong lens systems (III)

- search a best-fit model by χ^2 minimization

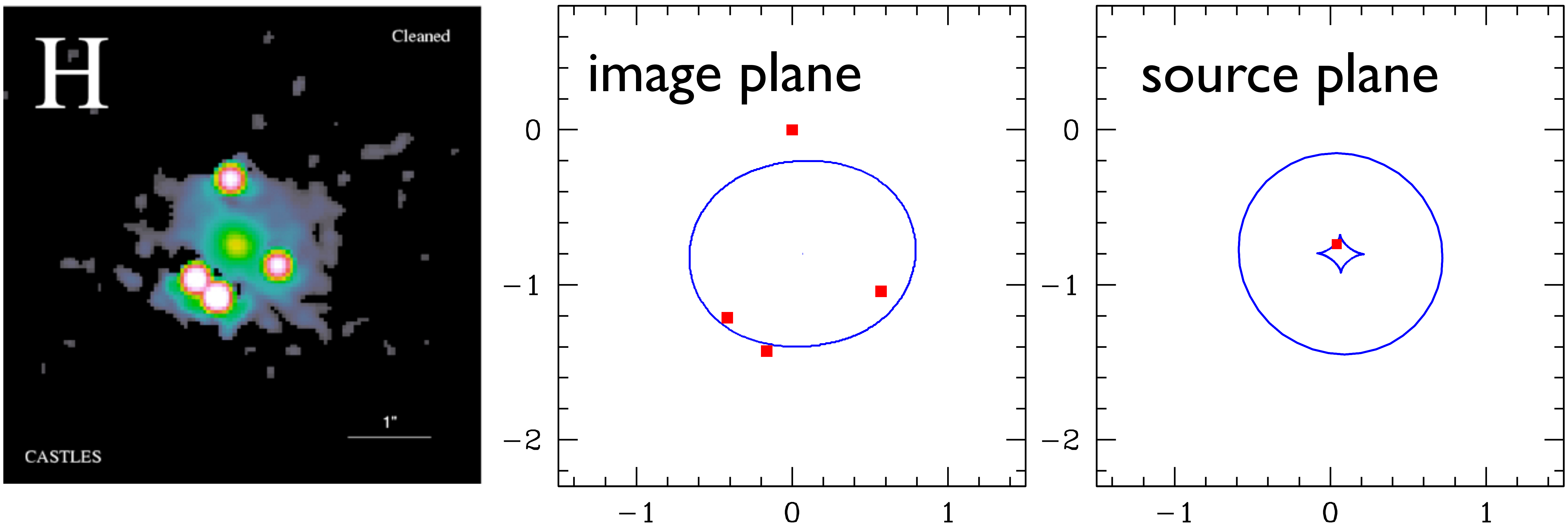
$$\chi^2 = \sum_i \frac{\left|\vec{\theta}_{i,\mathrm{model}} - \vec{\theta}_{i,\mathrm{obs}}\right|^2}{\sigma_{\theta_i}^2} + \sum_{ij} \frac{(\Delta m_{ij,\mathrm{model}} - \Delta m_{ij,\mathrm{obs}})^2}{\sigma_{\Delta m_{ij}}^2}$$

- [advanced] trick: source plane χ^2 minimization

$$\vec{\theta}_{i,\mathrm{model}} - \vec{\theta}_{i,\mathrm{obs}} \approx A^{-1}(\vec{\theta}_{i,\mathrm{obs}}) \left[\vec{\beta}_{\mathrm{model}} - \vec{\beta}(\vec{\theta}_{i,\mathrm{obs}})\right]$$

computation much faster, but # of images can be wrong (need cross-check)

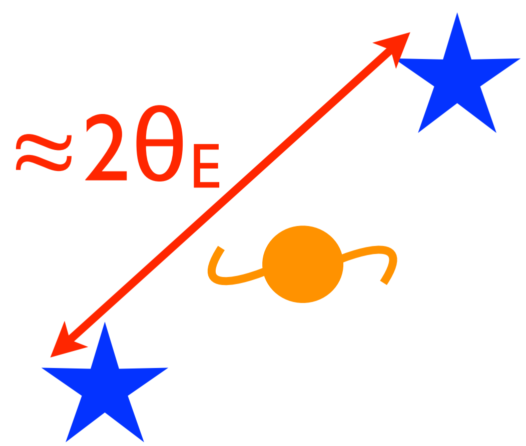
Modeling strong lens systems (IV)



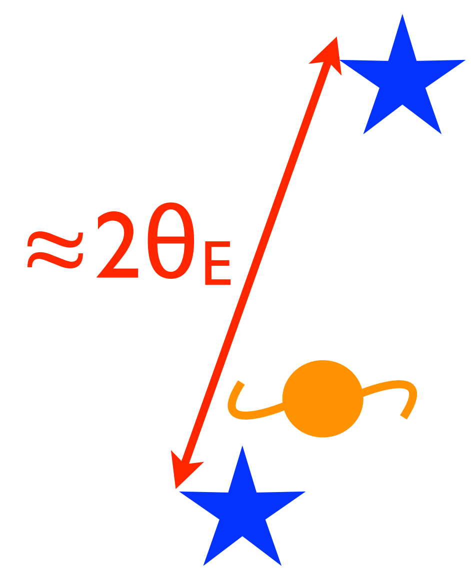
- result obtained using *glafic*
- best-fit model has χ^2/d.o.f = 6.4/4

What does strong lens measure? (I)

- angular separation between images $\approx 2\theta_E$



'symmetric' configuration



'asymmetric' configuration

- therefore, multiple images provides good measurements of the Einstein radius θ_E

simulated by *glafic*

Image separation and Einstein radius

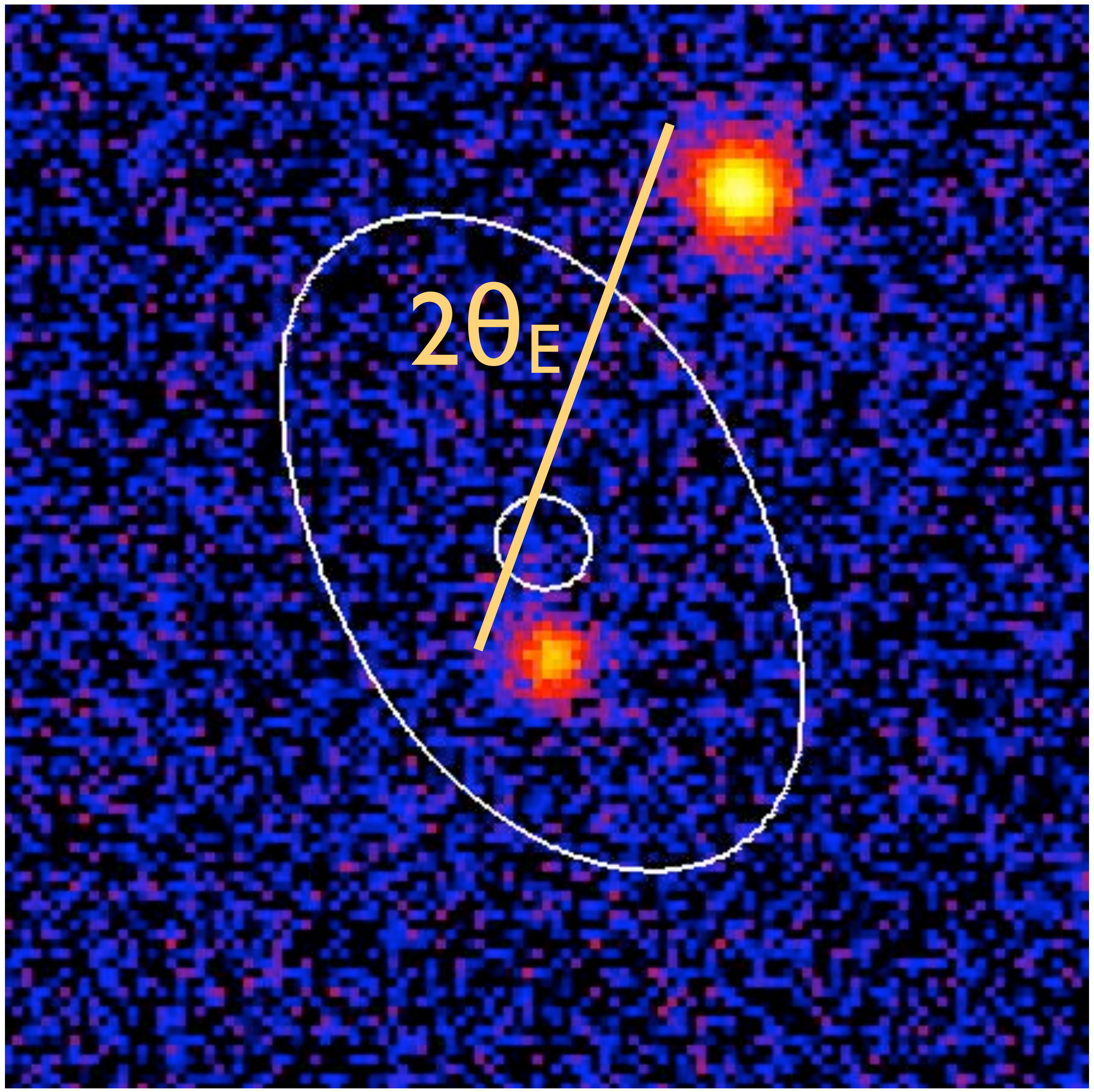
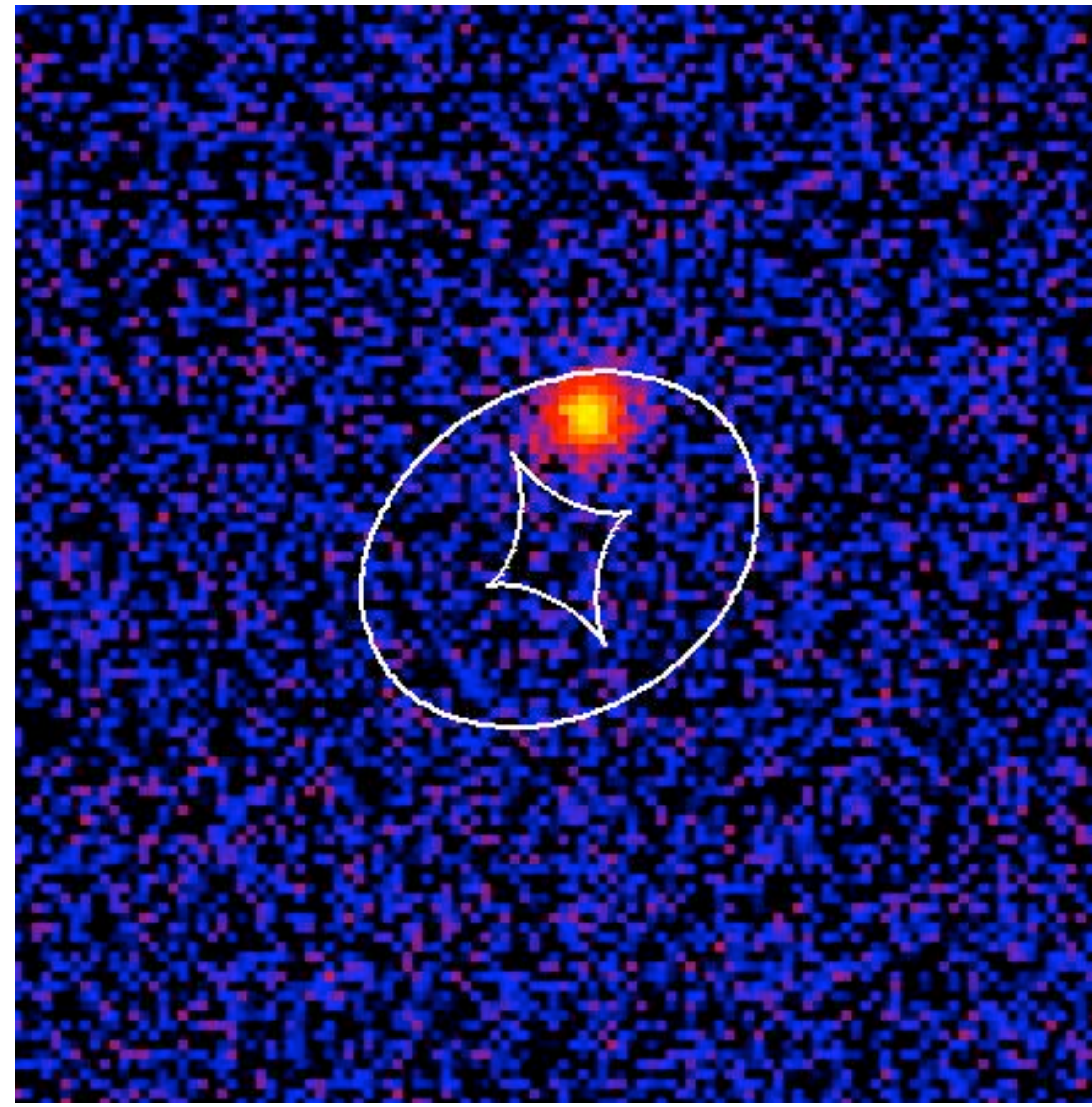


image plane
(critical curves)



source plane
(caustics)

simulated by *glafic*

Image separation and Einstein radius

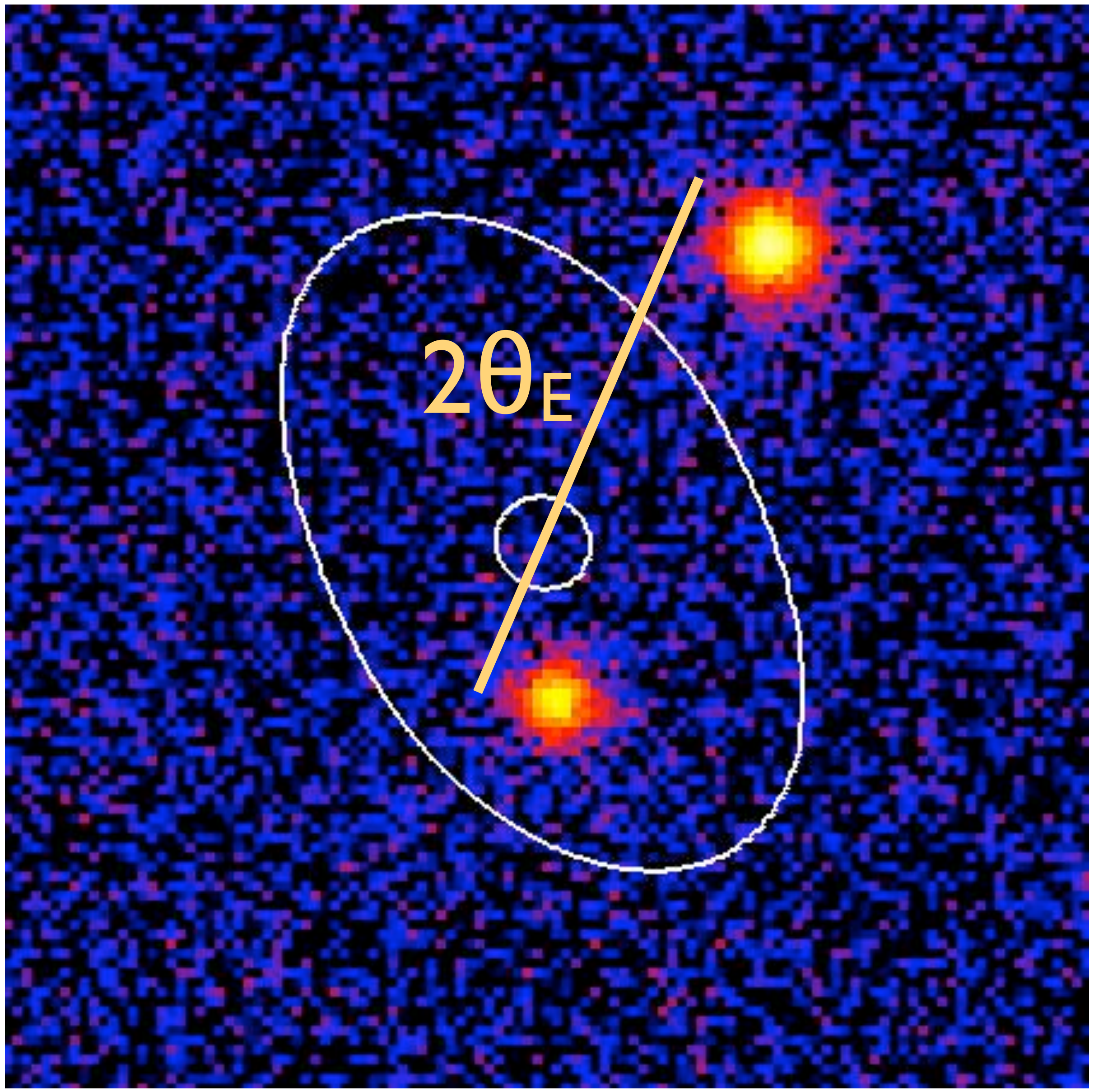
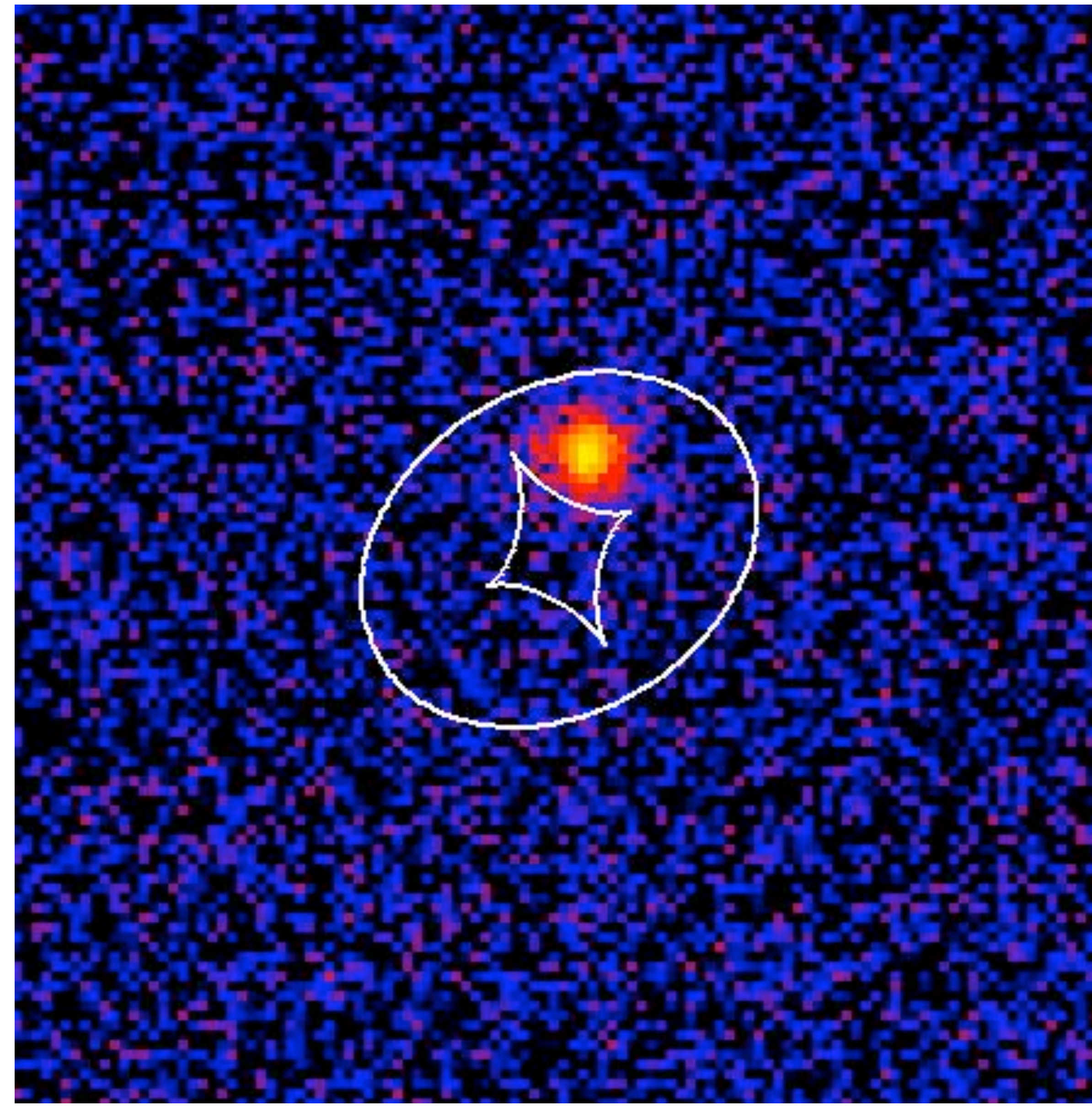


image plane
(critical curves)



source plane
(caustics)

simulated by *glafic*

Image separation and Einstein radius

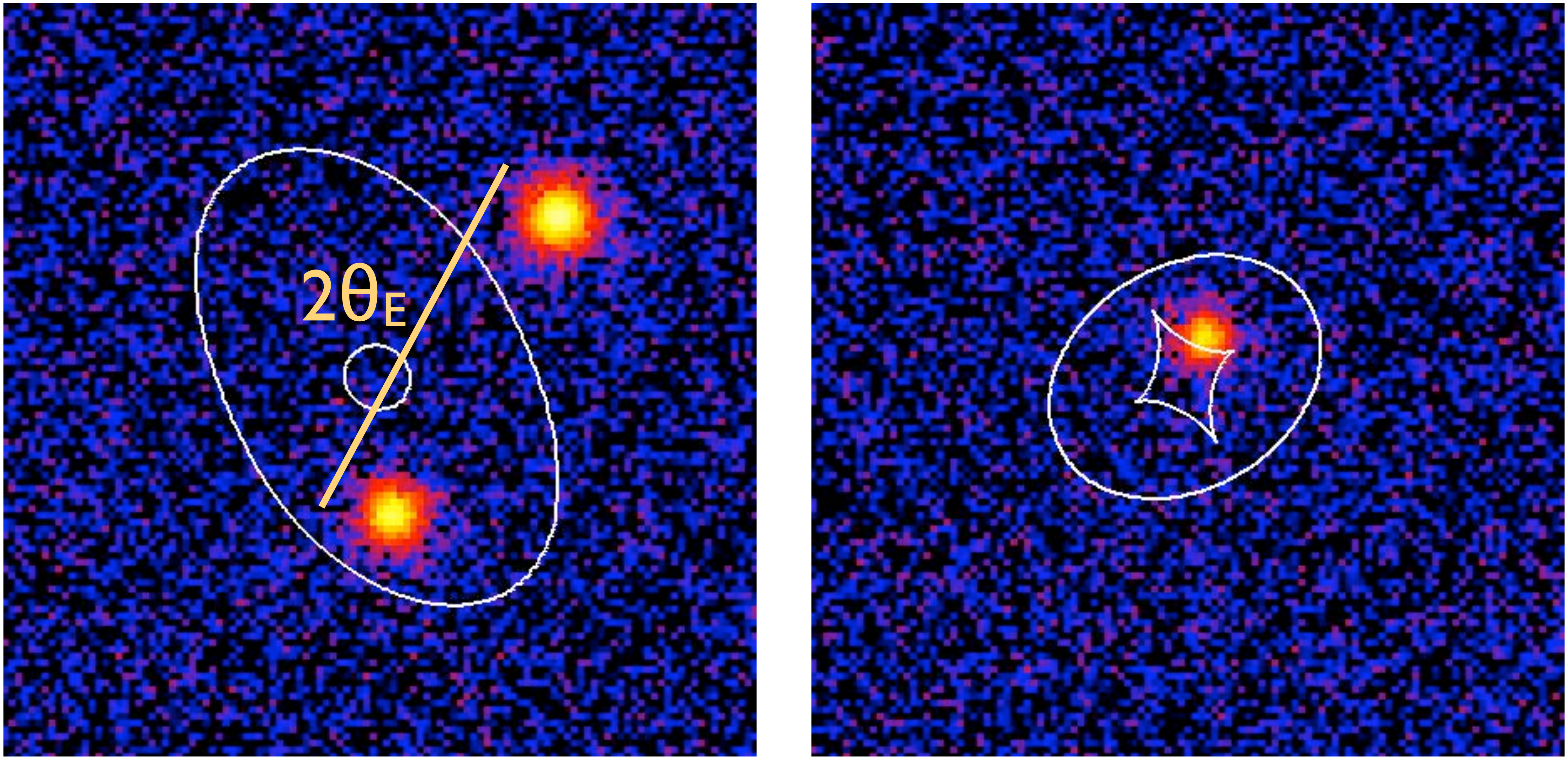


image plane
(critical curves)

source plane
(caustics)

Image separation and Einstein radius

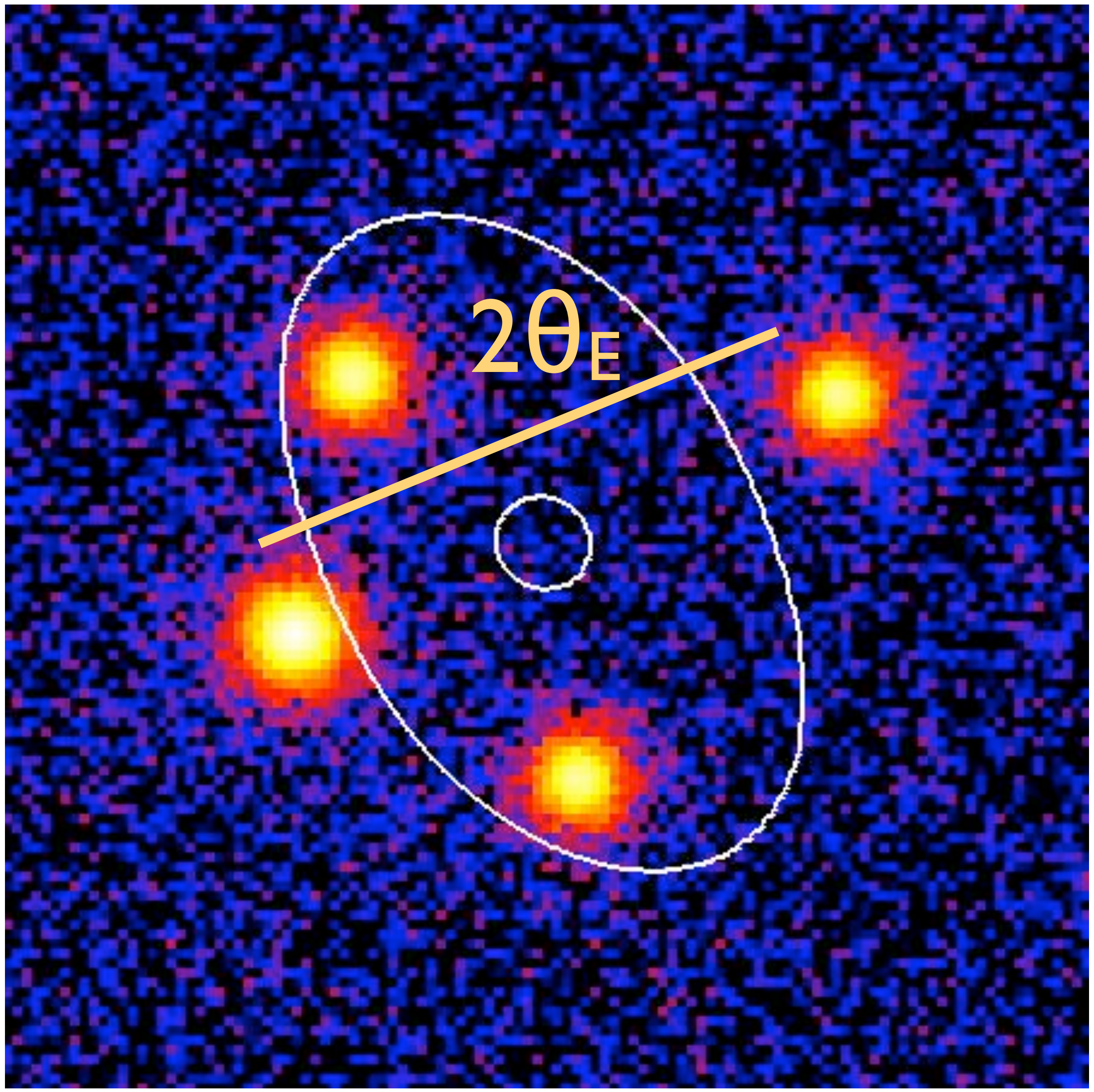
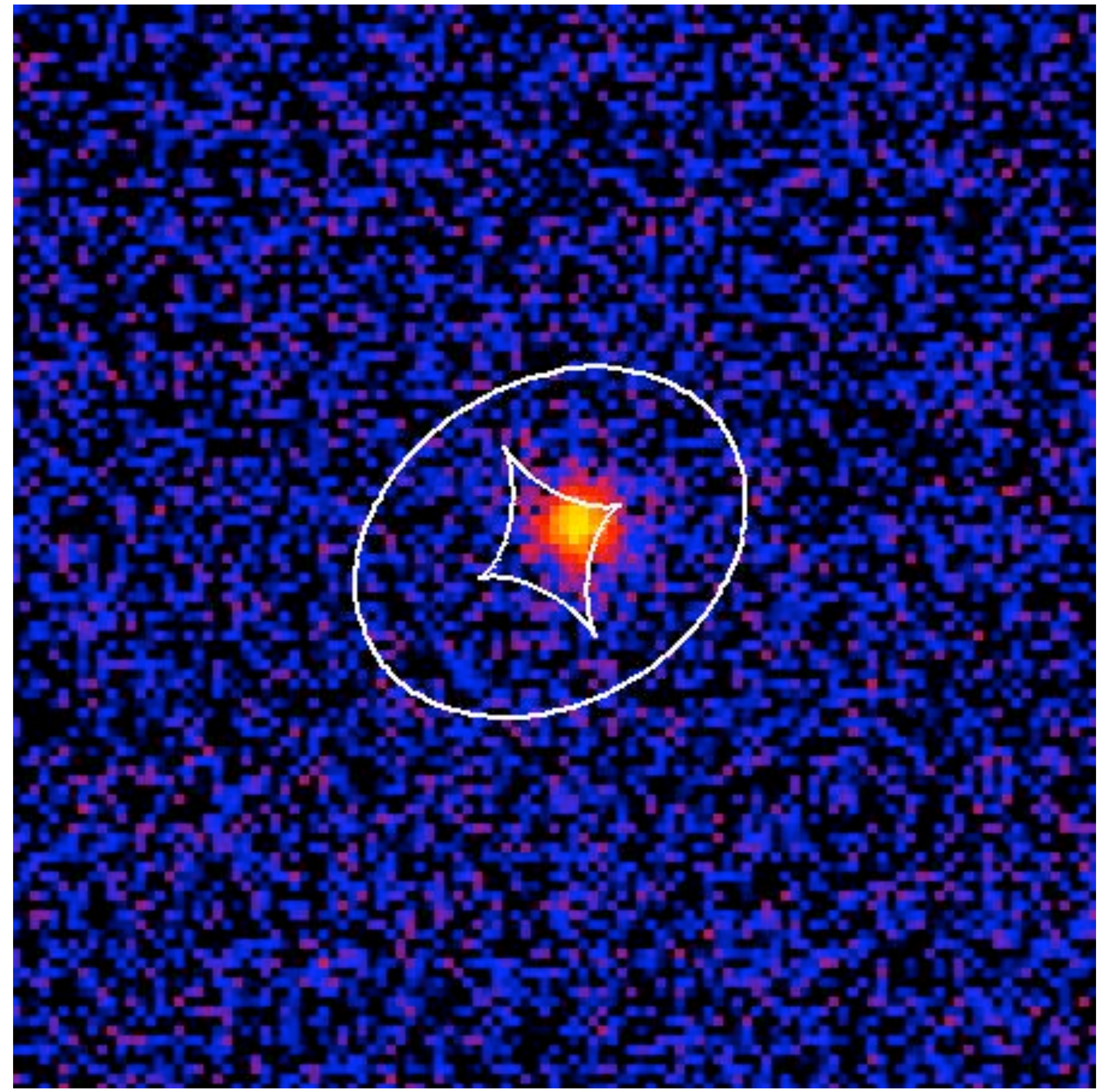


image plane
(critical curves)



source plane
(caustics)

simulated by *glafic*

Image separation and Einstein radius

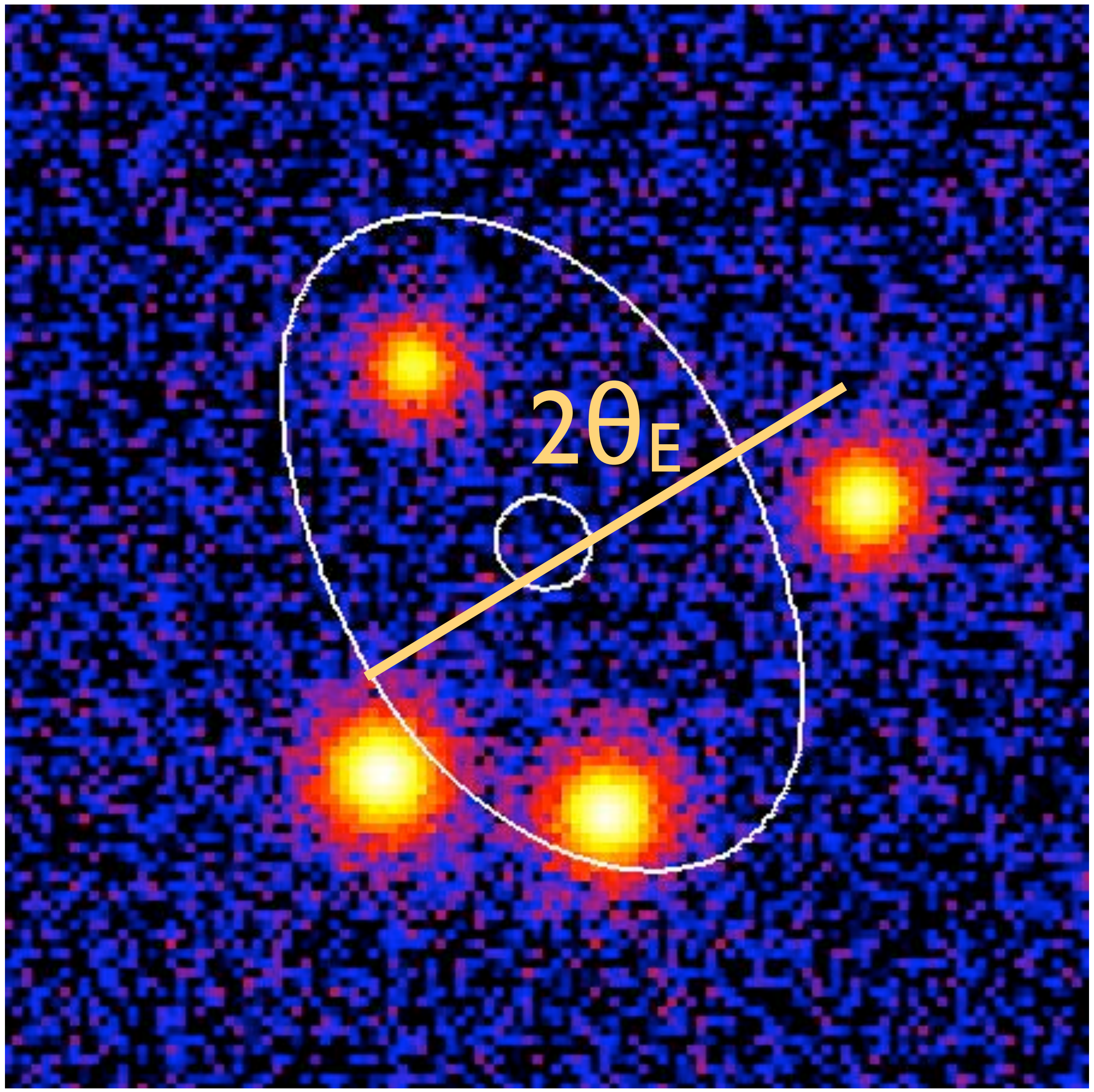
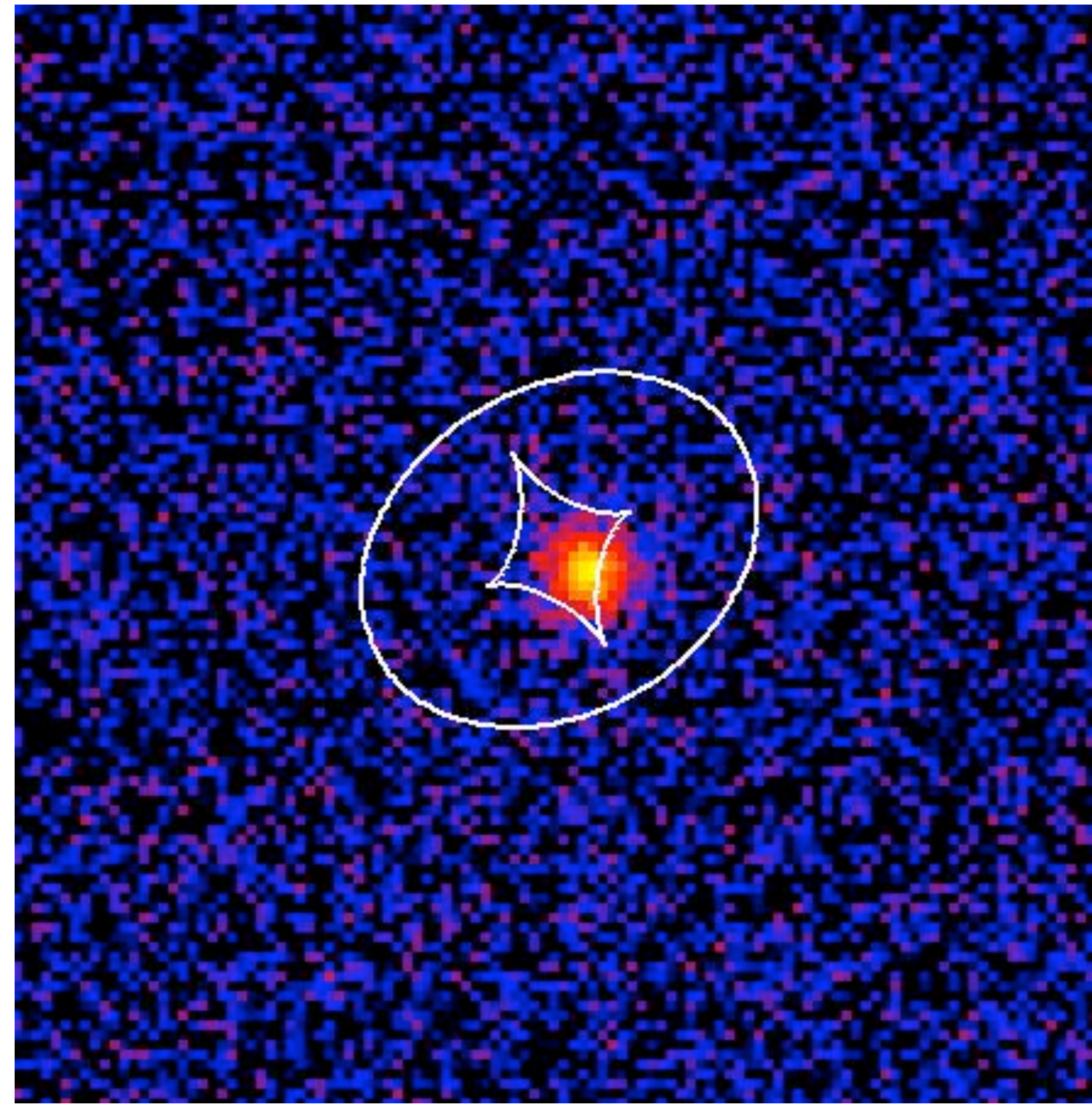


image plane
(critical curves)



source plane
(caustics)

simulated by *glafic*

Image separation and Einstein radius

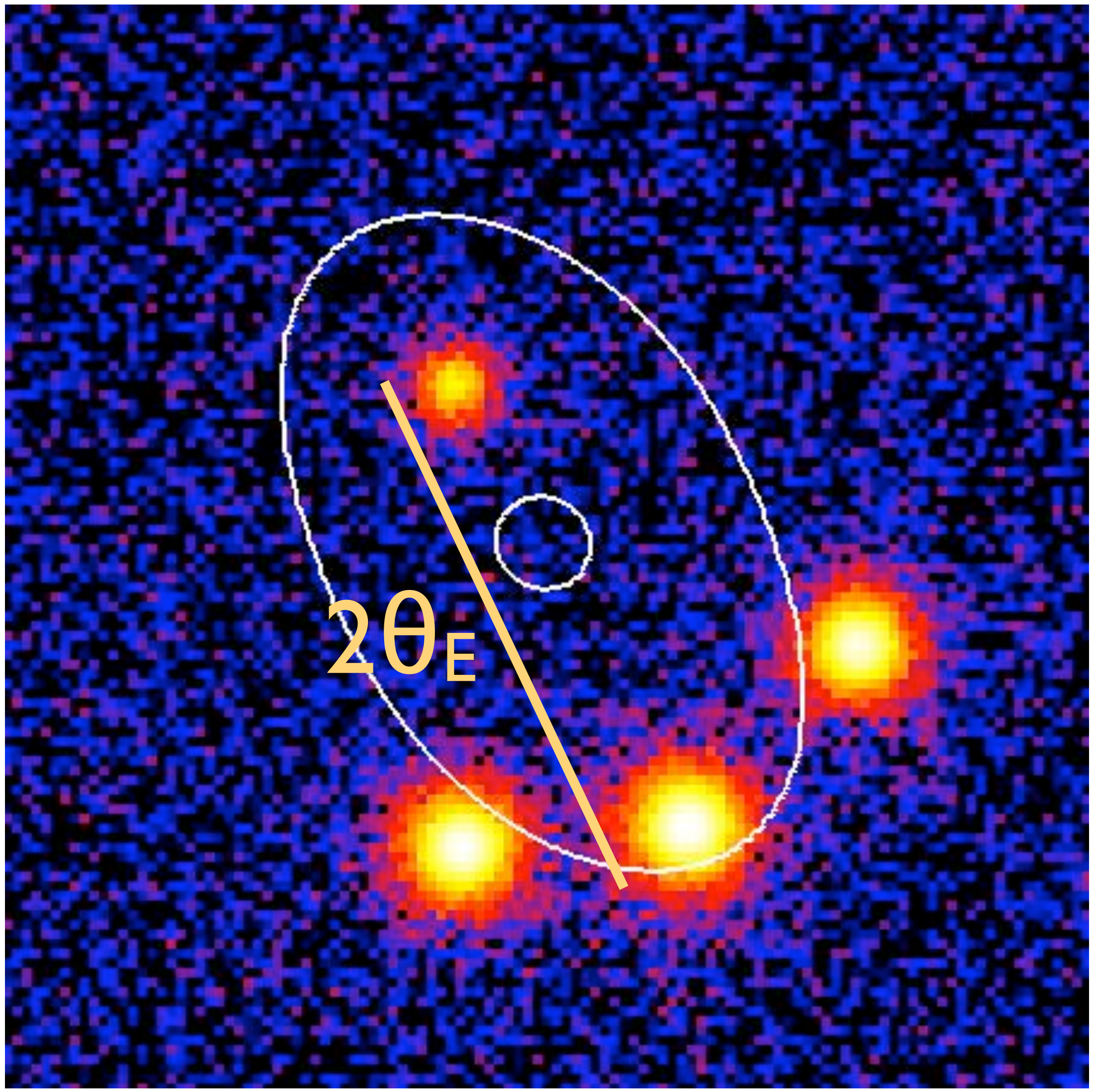
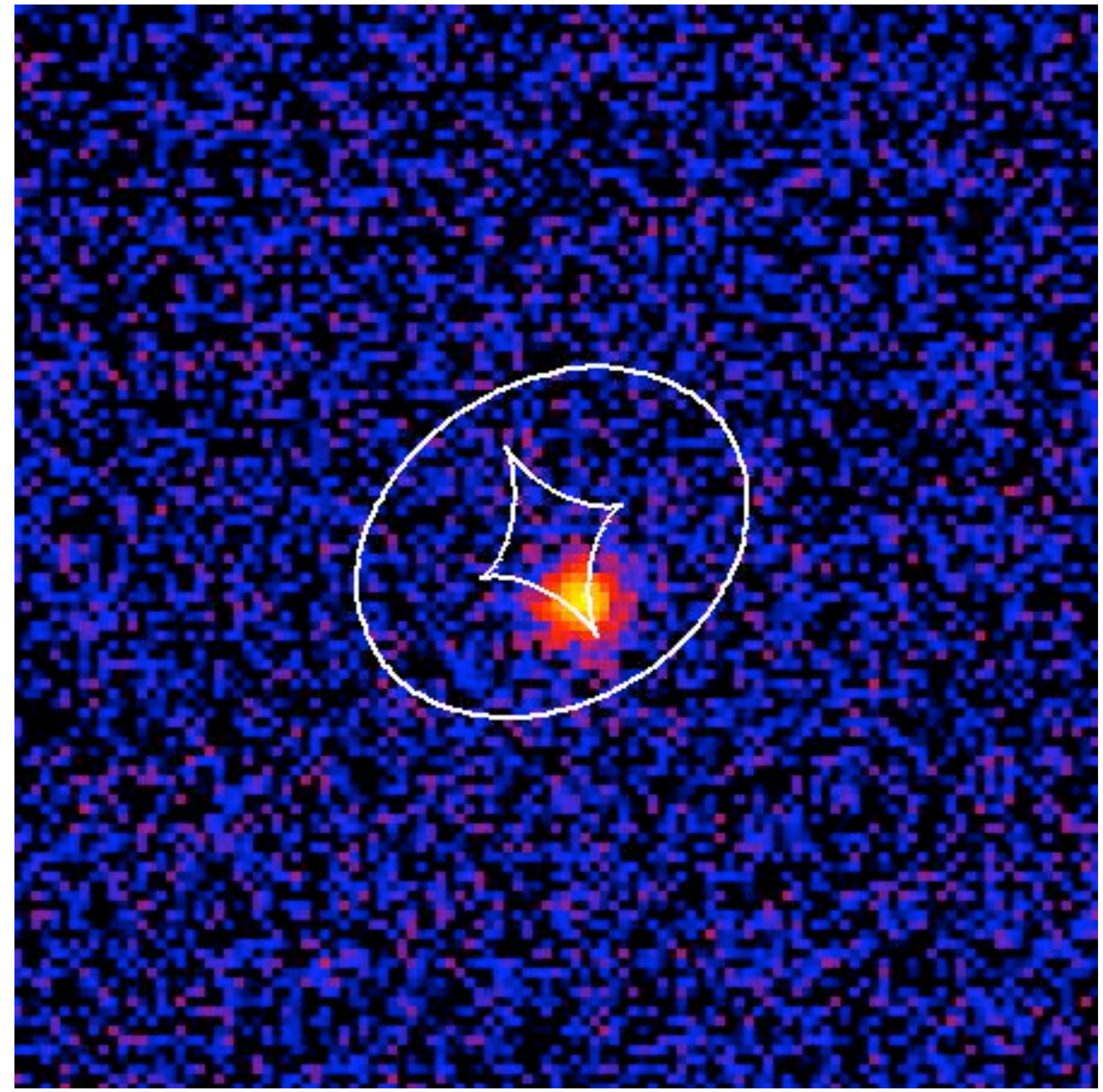


image plane
(critical curves)



source plane
(caustics)

Image separation and Einstein radius

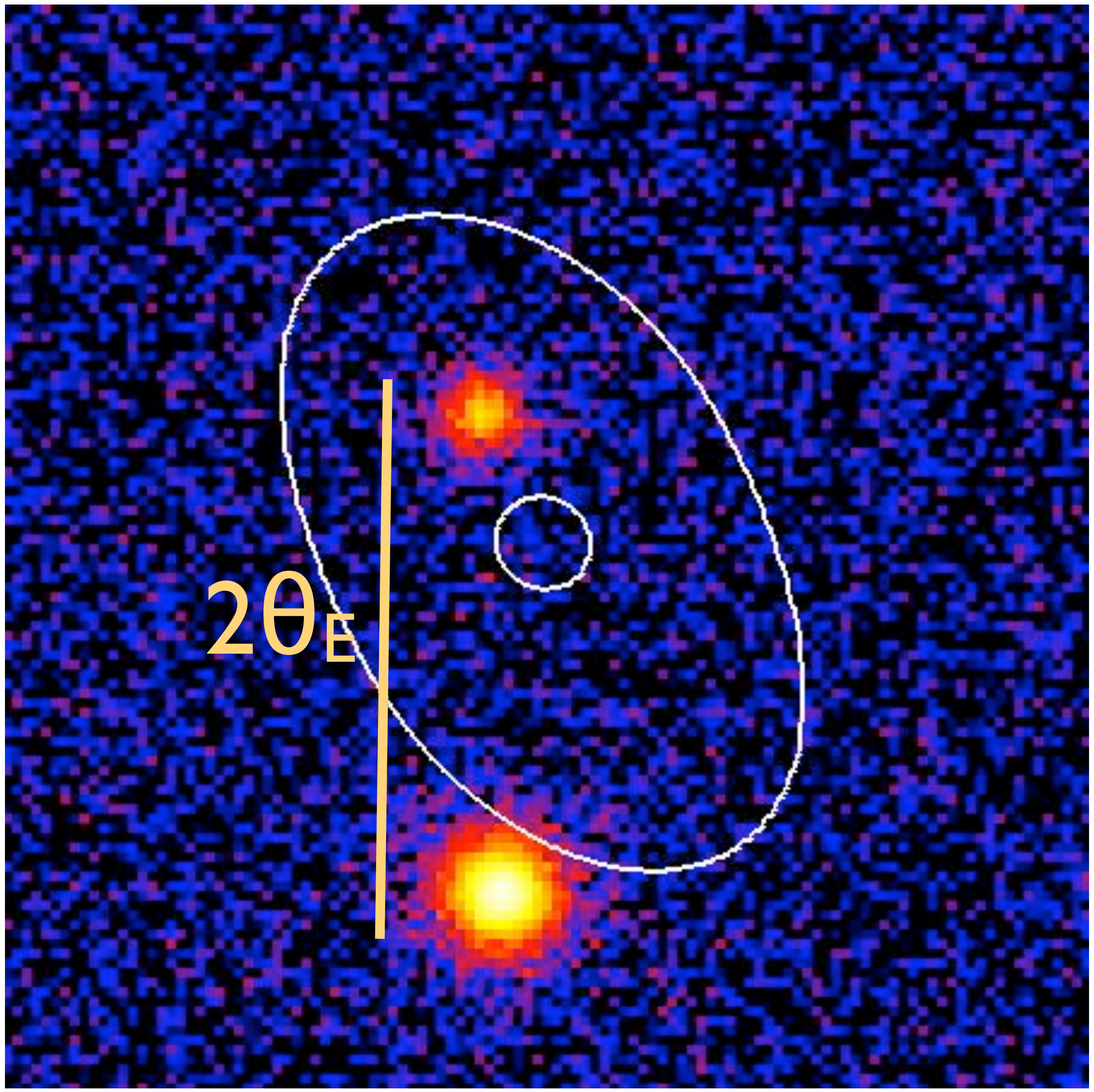
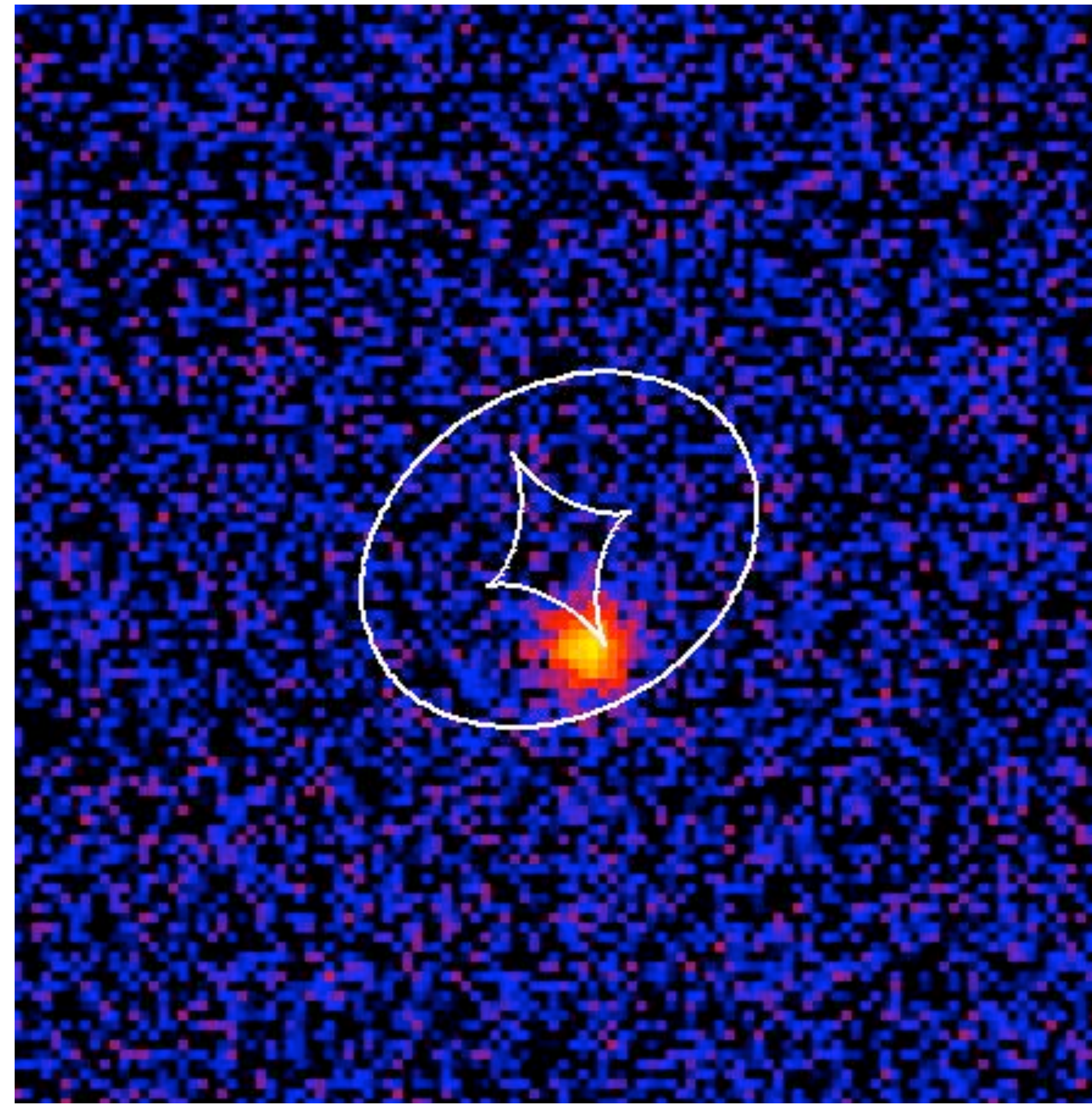


image plane
(critical curves)



source plane
(caustics)

simulated by *glafic*

Image separation and Einstein radius

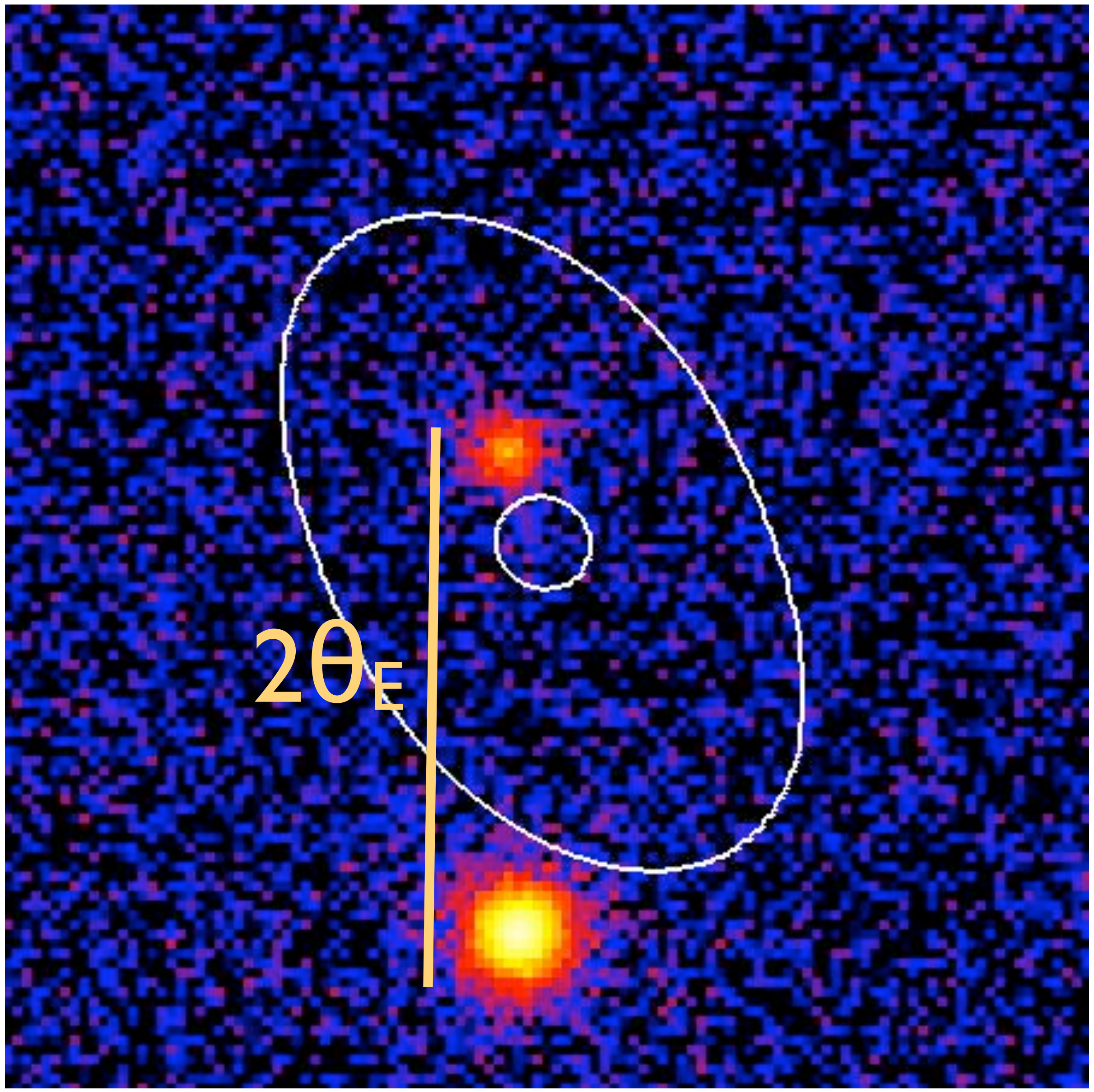
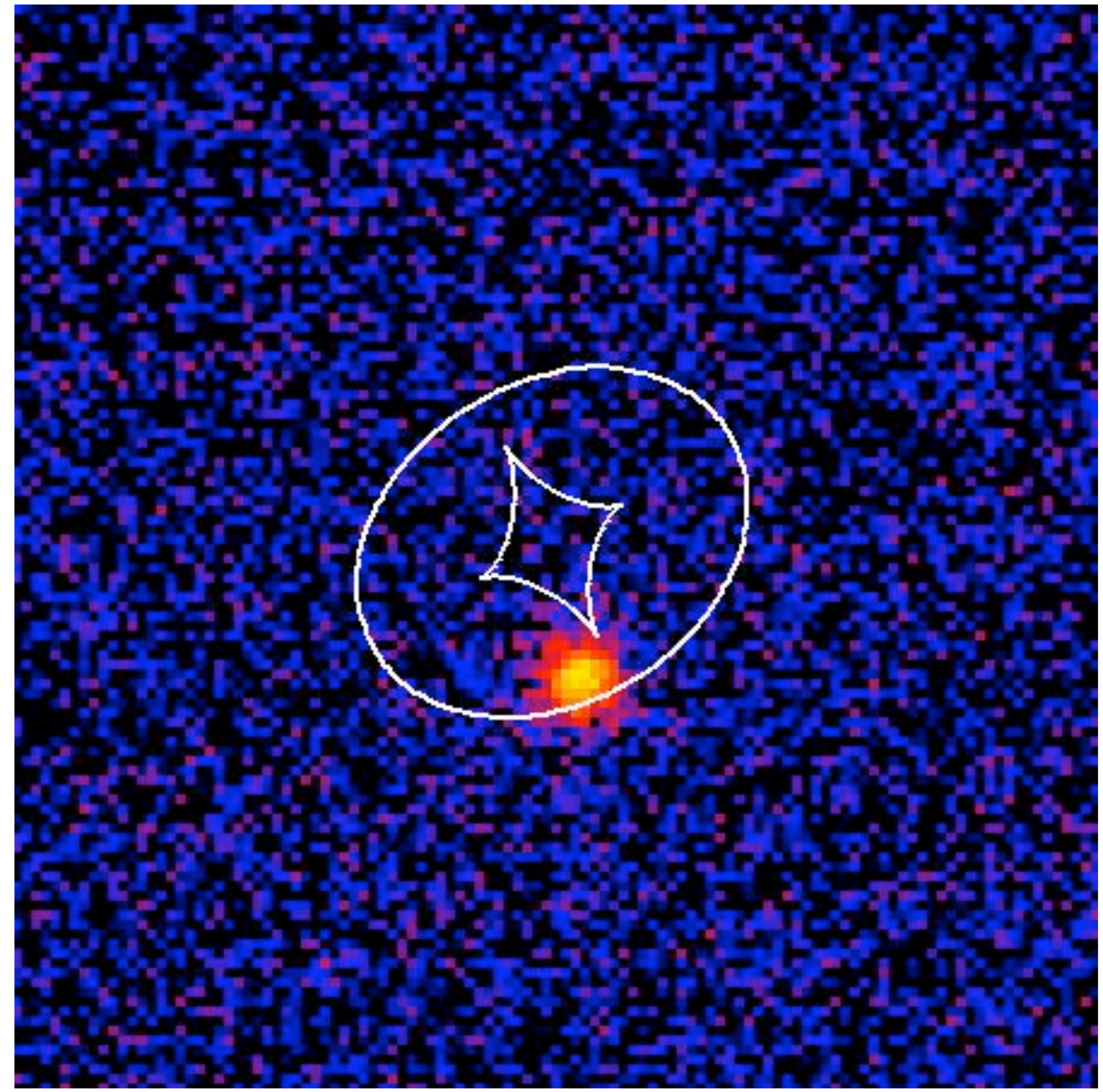


image plane
(critical curves)



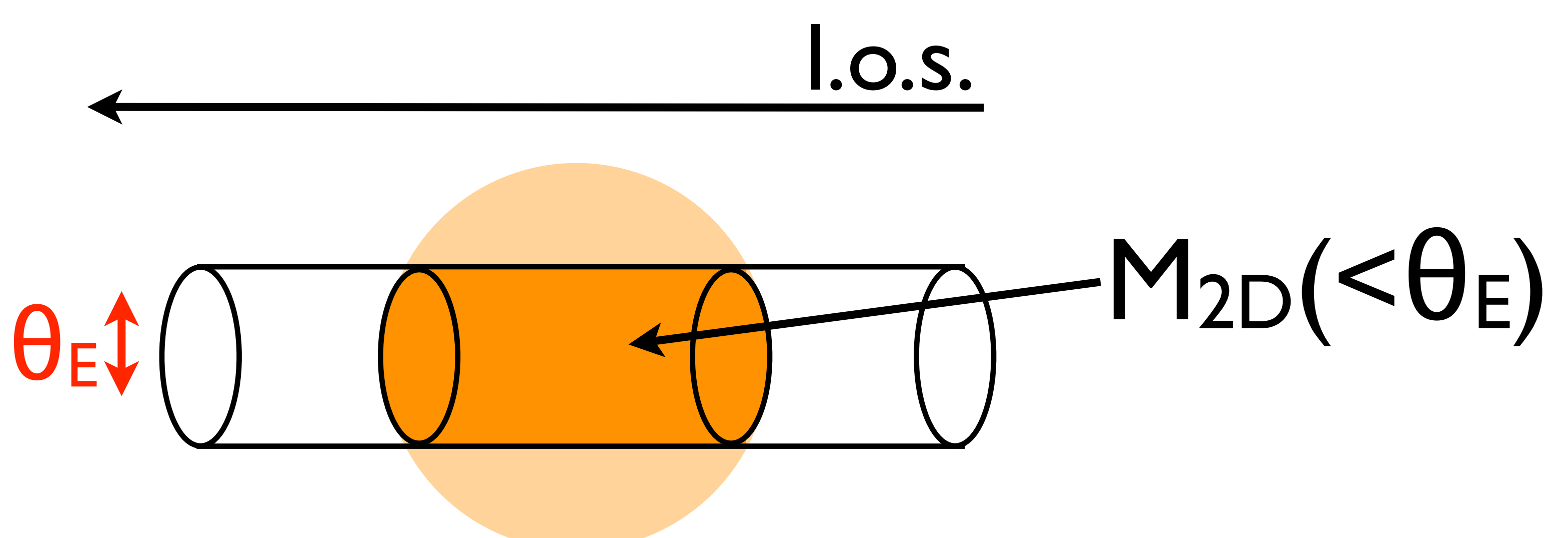
source plane
(caustics)

What does strong lens measure? (II)

- recall: the Einstein radius θ_E is determined by

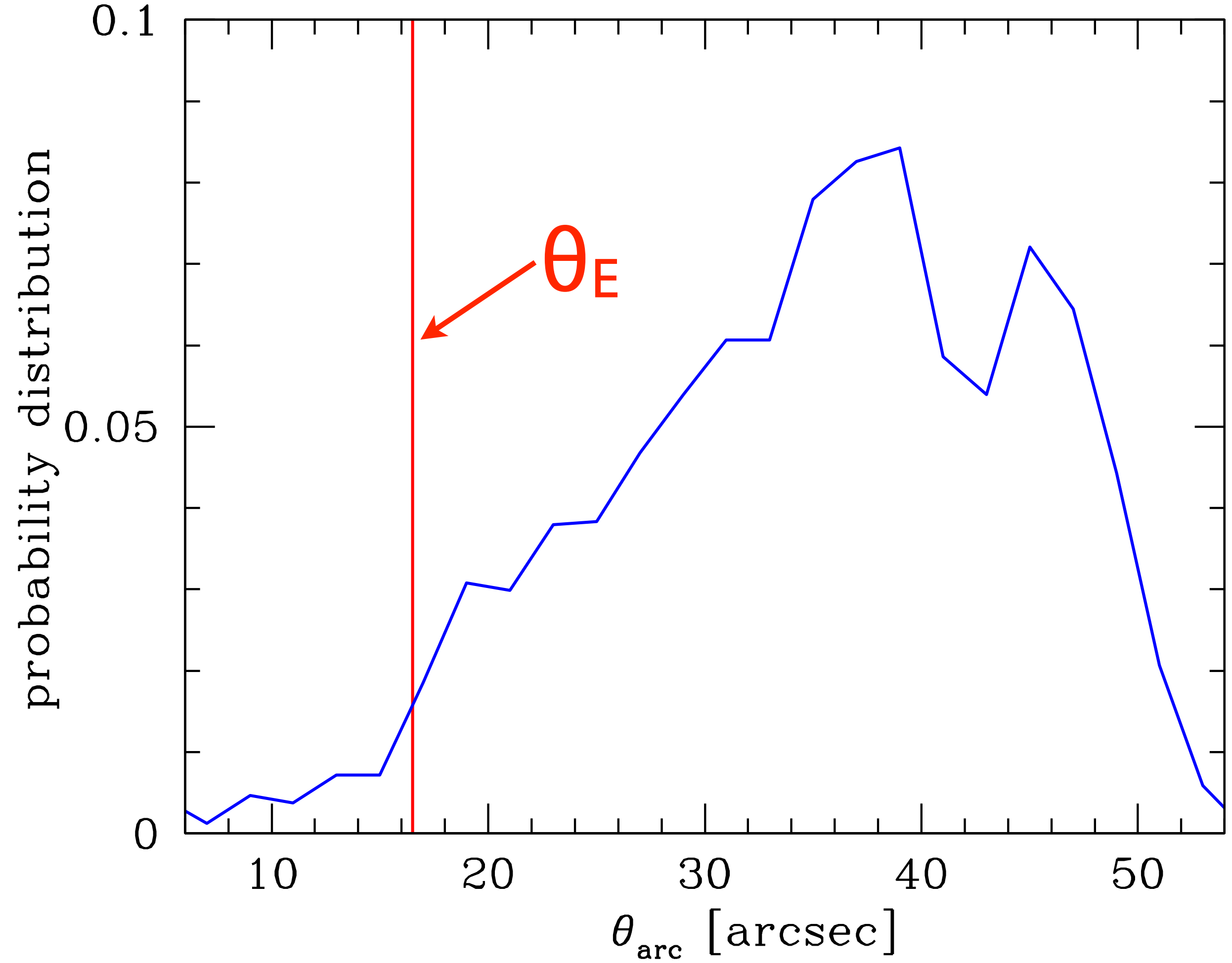
$$1 = \bar{\kappa}(< \theta_{\rm E}) = \frac{M_{2D}(< \theta_{\rm E})}{\pi \theta_{\rm E}^2 D_A^2(z_l) \Sigma_{\rm cr}}$$

→ strong lensing well constrains projected 2D mass within Einstein radius, $M_{2D}(< \theta_E)$



Note: position of arcs

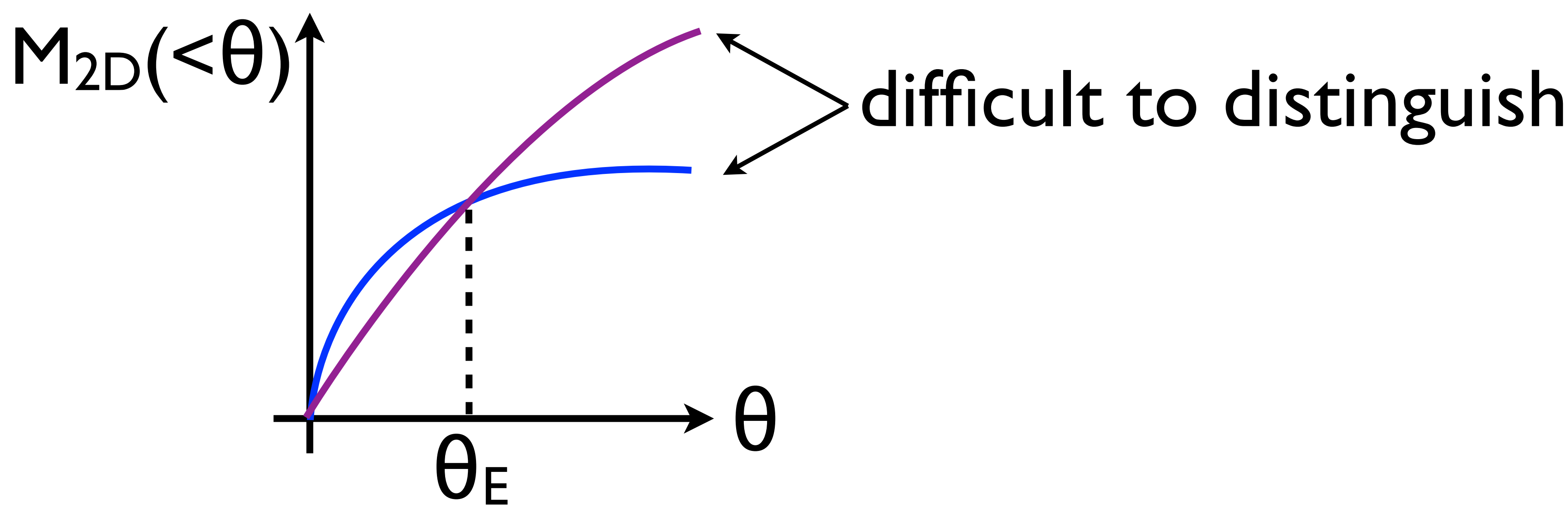
- sometimes people take positions of arcs θ_{arc} and assume $\theta_{arc}=\theta_E$
- this can be quite wrong, because arcs can be produced in asymmetric configurations, and arcs are produced preferentially along the major axis...



distribution of θ_{arc} for massive cluster with e=0.4 (simulated by *glafic*)

What does strong lens measure? (III)

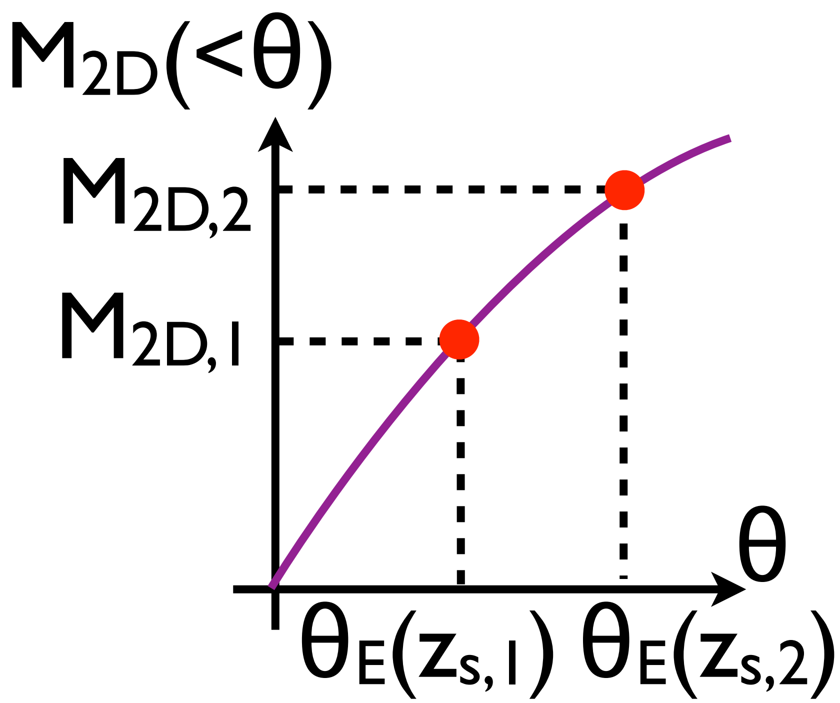
- on the other hand, radial density profile is usually not very well constrained



- possible ways to constrain radial profiles
  (1) strong lenses with different z_s
  (2) more constraints (time delays, arcs, ...)
  (3) complementary mass probes (velocity dispersion, weak lensing, ...)

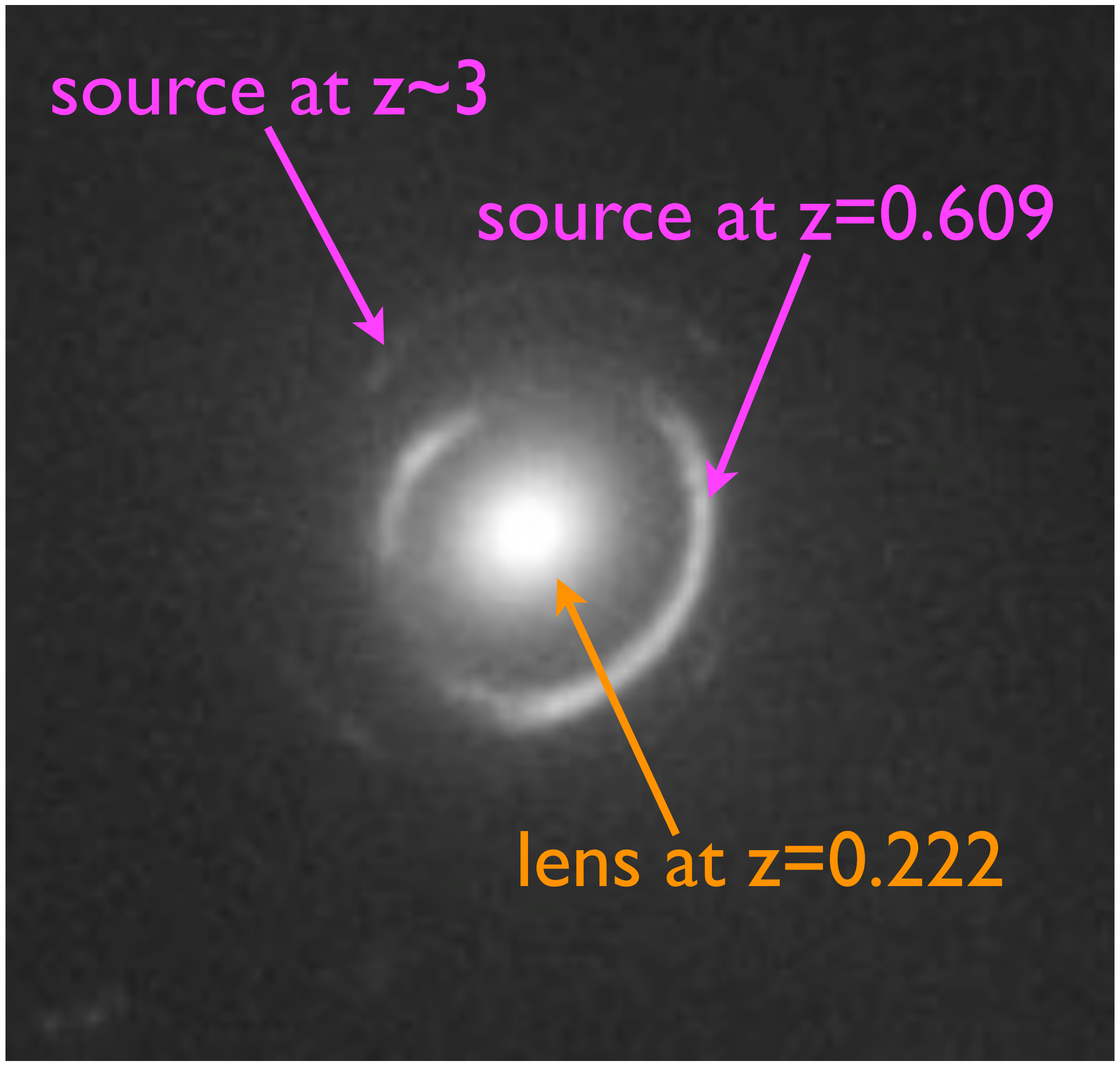
Multiple z_s

- multiple strongly lens systems in a same lens constrains enclosed masses at different radii

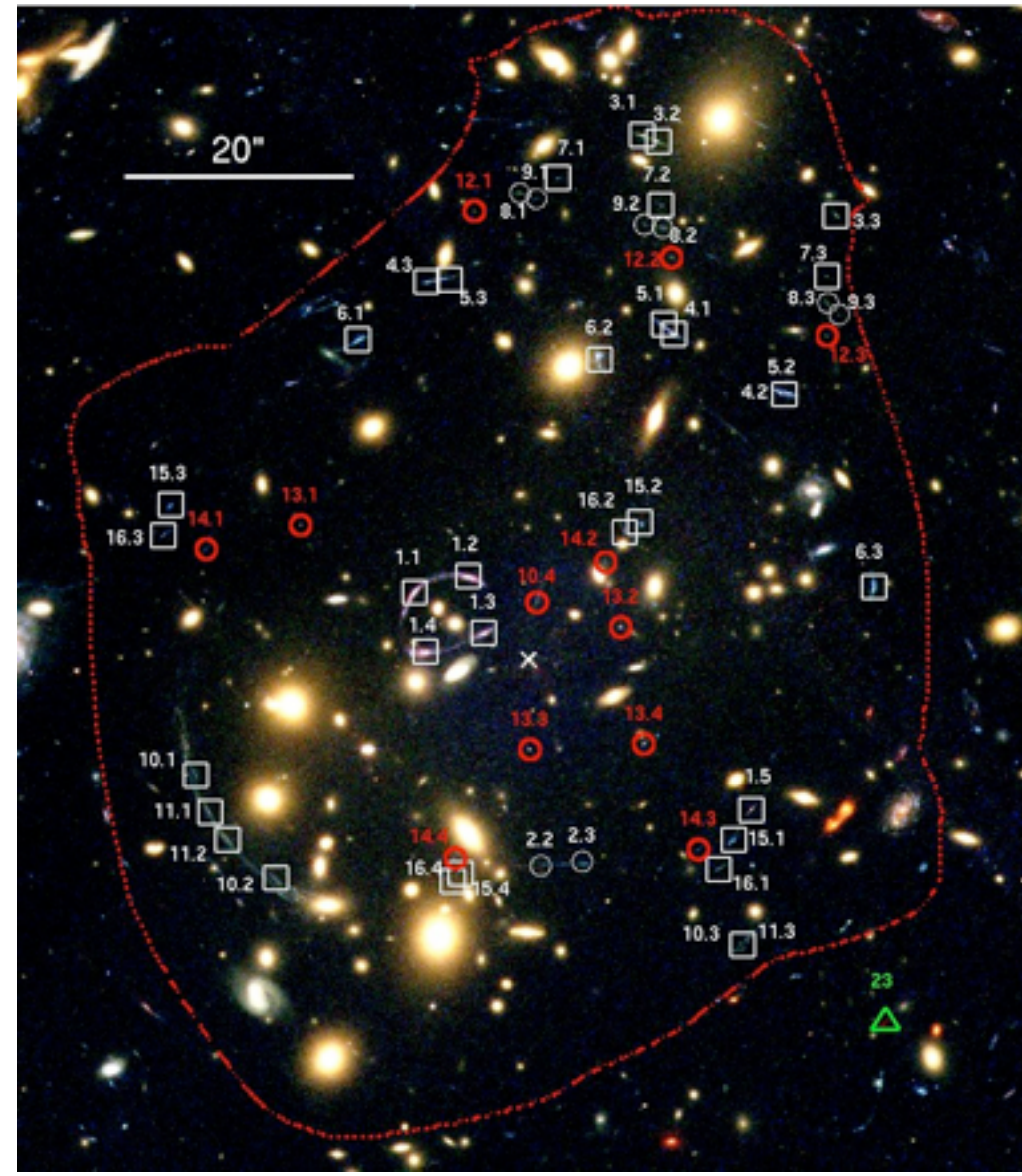


- useful to constrain radial profiles, and possibly cosmological parameters as well, particularly if combined with other radial profile probes

Multiple z_s: examples



SDSSJ0946+1006
'double Einstein ring'
(Gavazzi et al. 2008)

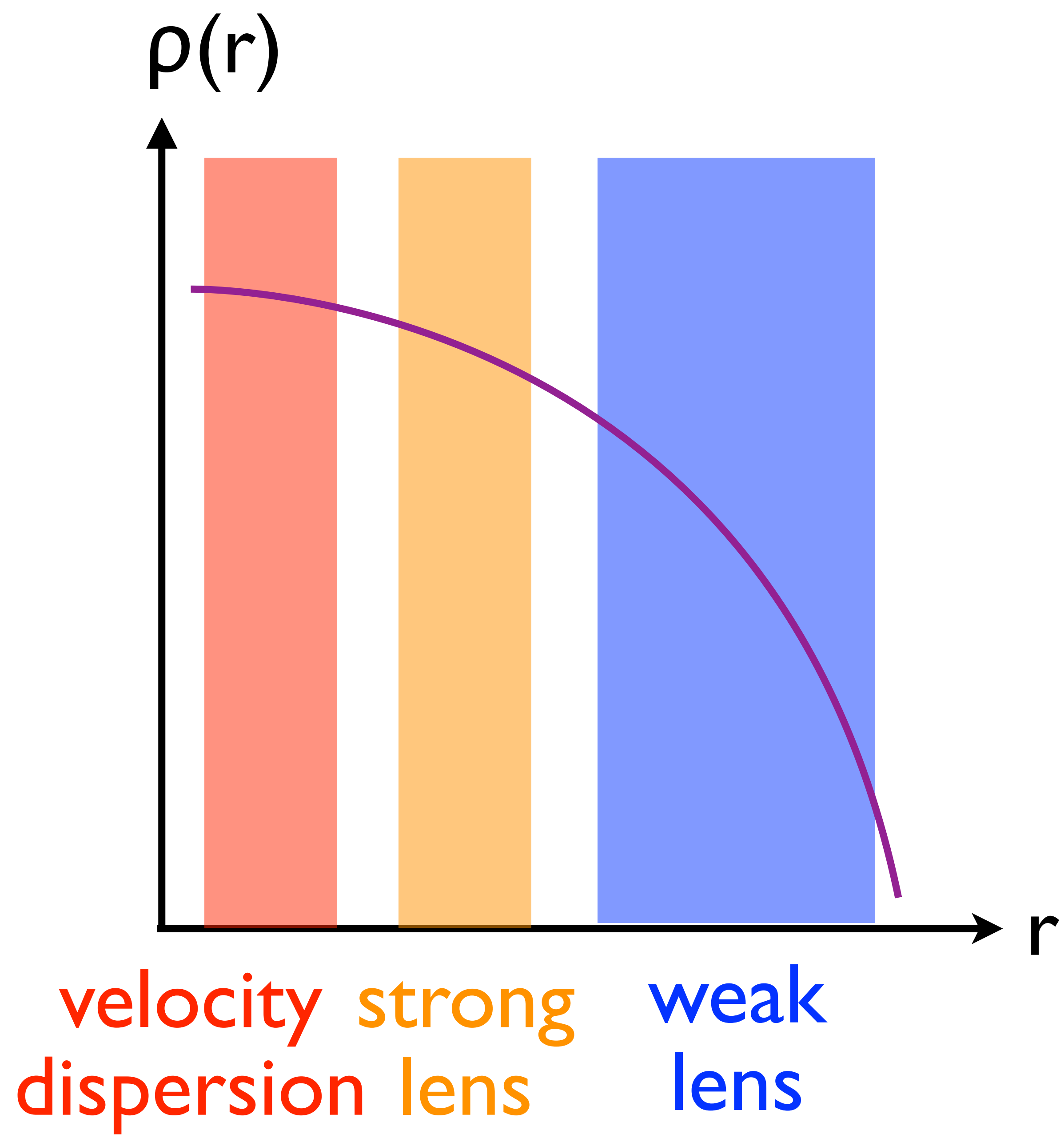


Abell 1703
(Richard et al. 2009;
Oguri et al. 2009)

Multiple probes

- velocity dispersion
→ probe total mass
  at the very core

- weak lensing
→ probe outskirts
  of halos
  (next lecture!)



Summary

- solving lens equation in general is not easy (need sophisticated numerical techniques)
- behavior is easier to understand for circular symmetric cases
- strong lens systems essentially probe enclosed mass within the Einstein radius

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1. Introduction & basics of gravitational lensing
2. Strong lensing analysis
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