Applications of gravitational lensing in astrophysics and cosmology

I. Introduction & basics of gravitational lensing

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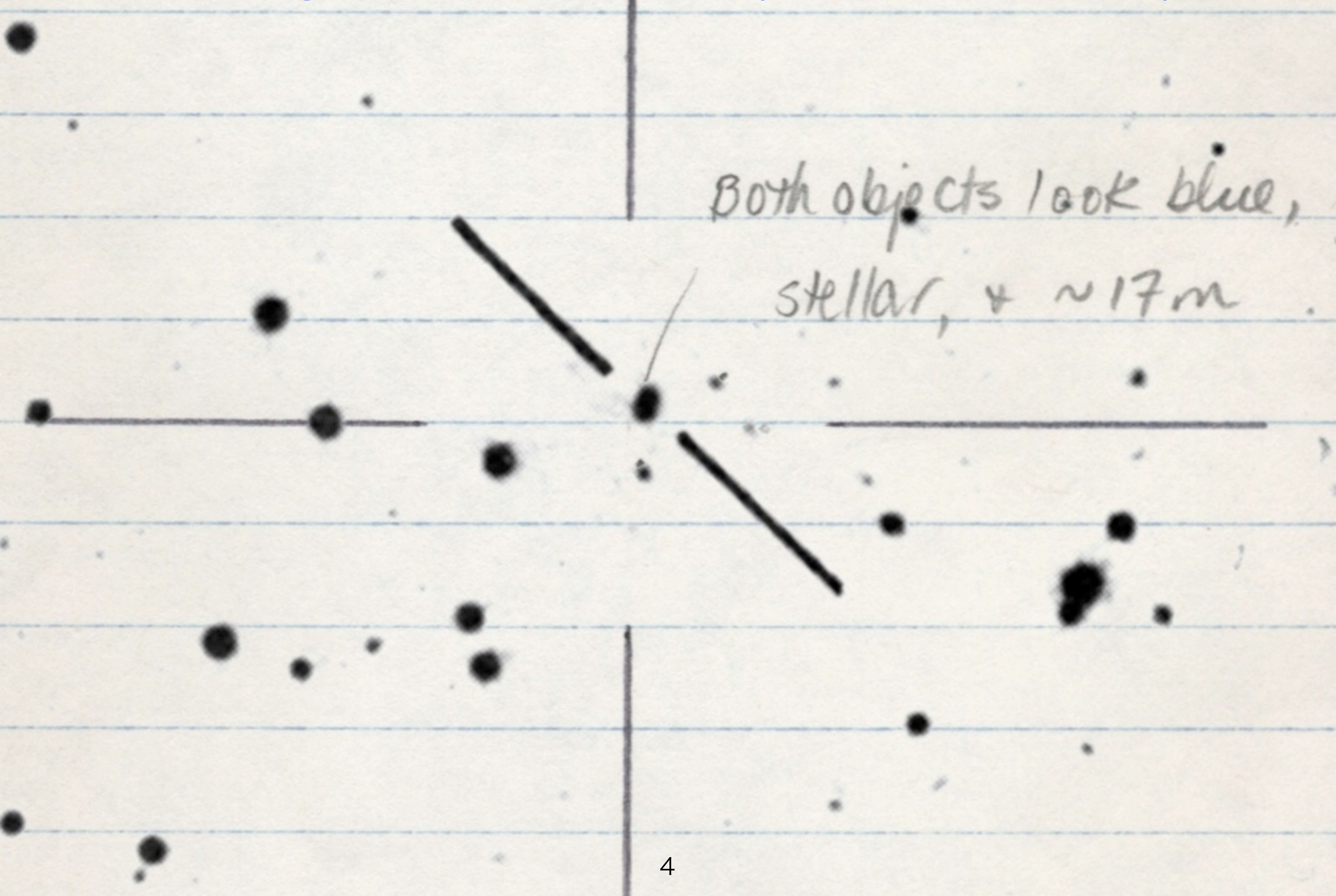
Contents

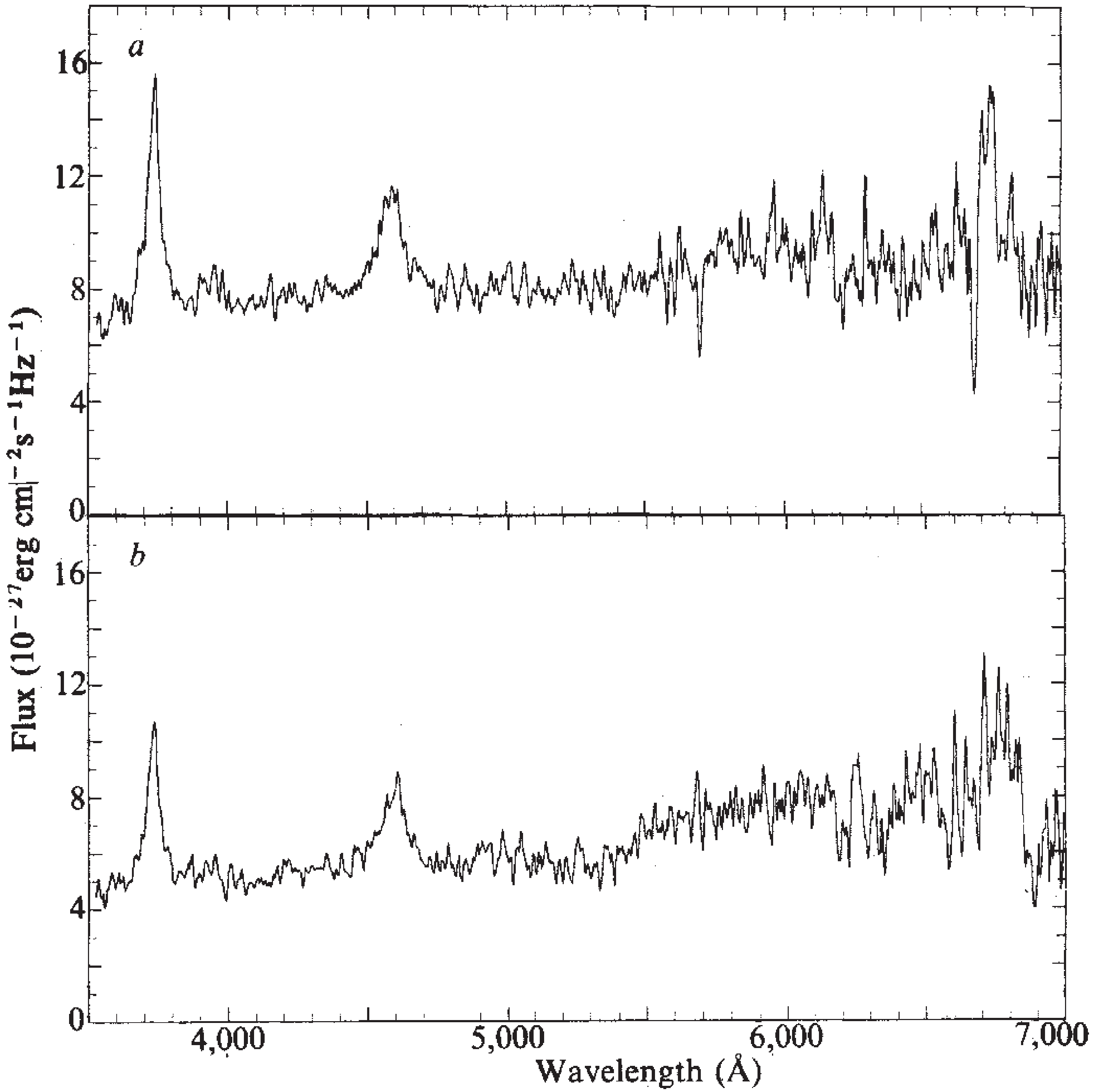
Intro: why gravitational lensing?

- the Universe is dominated by dark matter
  → lensing directly ‘see’ the invisible matter
- solid theoretical foundation: lensing phenomena robustly predicted for a given mass distribution from the first principle
  (general relativity, or even in modified gravity)
- pretty & impressive pictures!

First strong lens: Q0957+561 (Walsh et al. 1979)

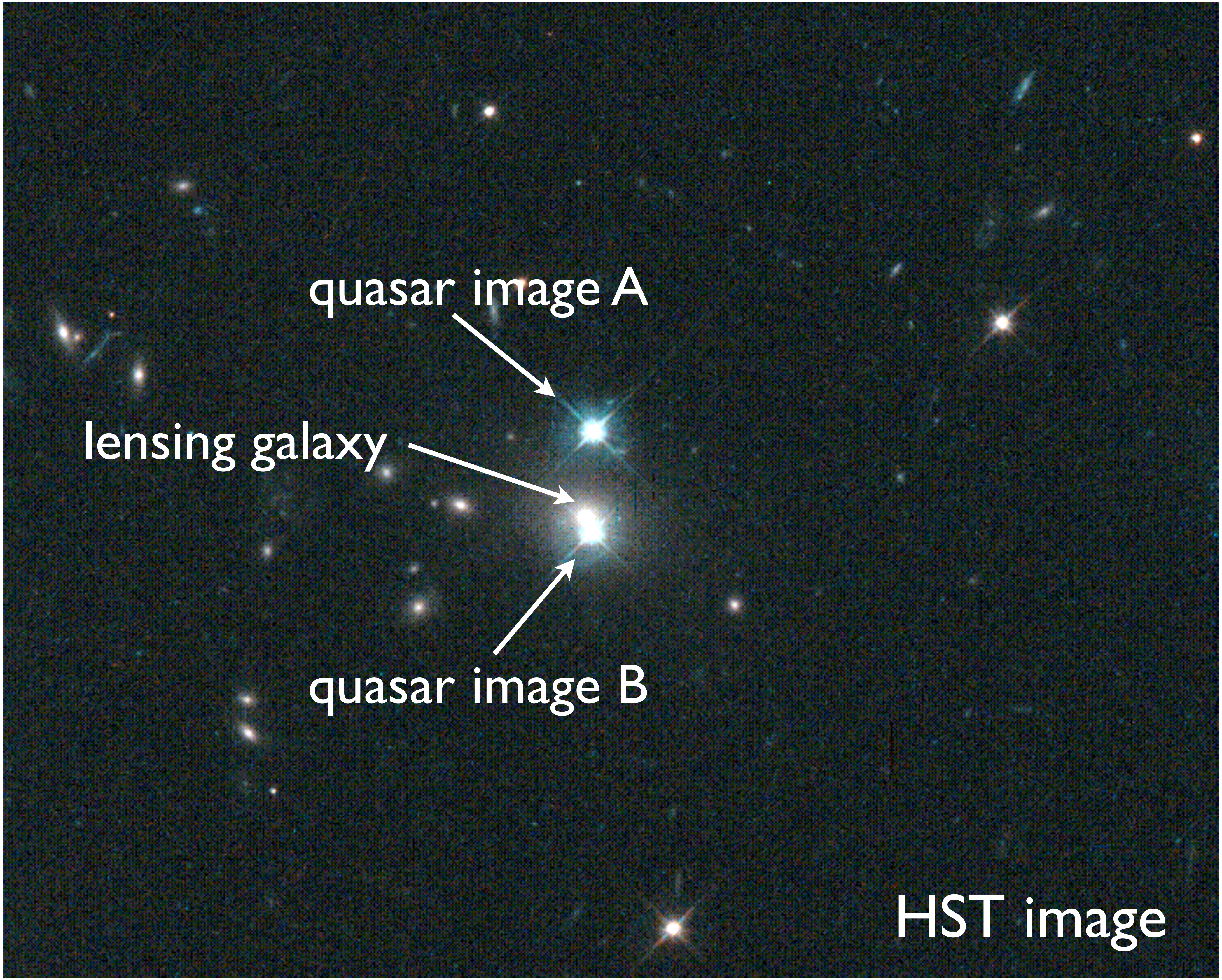


First strong lens: Q0957+561 (Walsh et al. 1979)

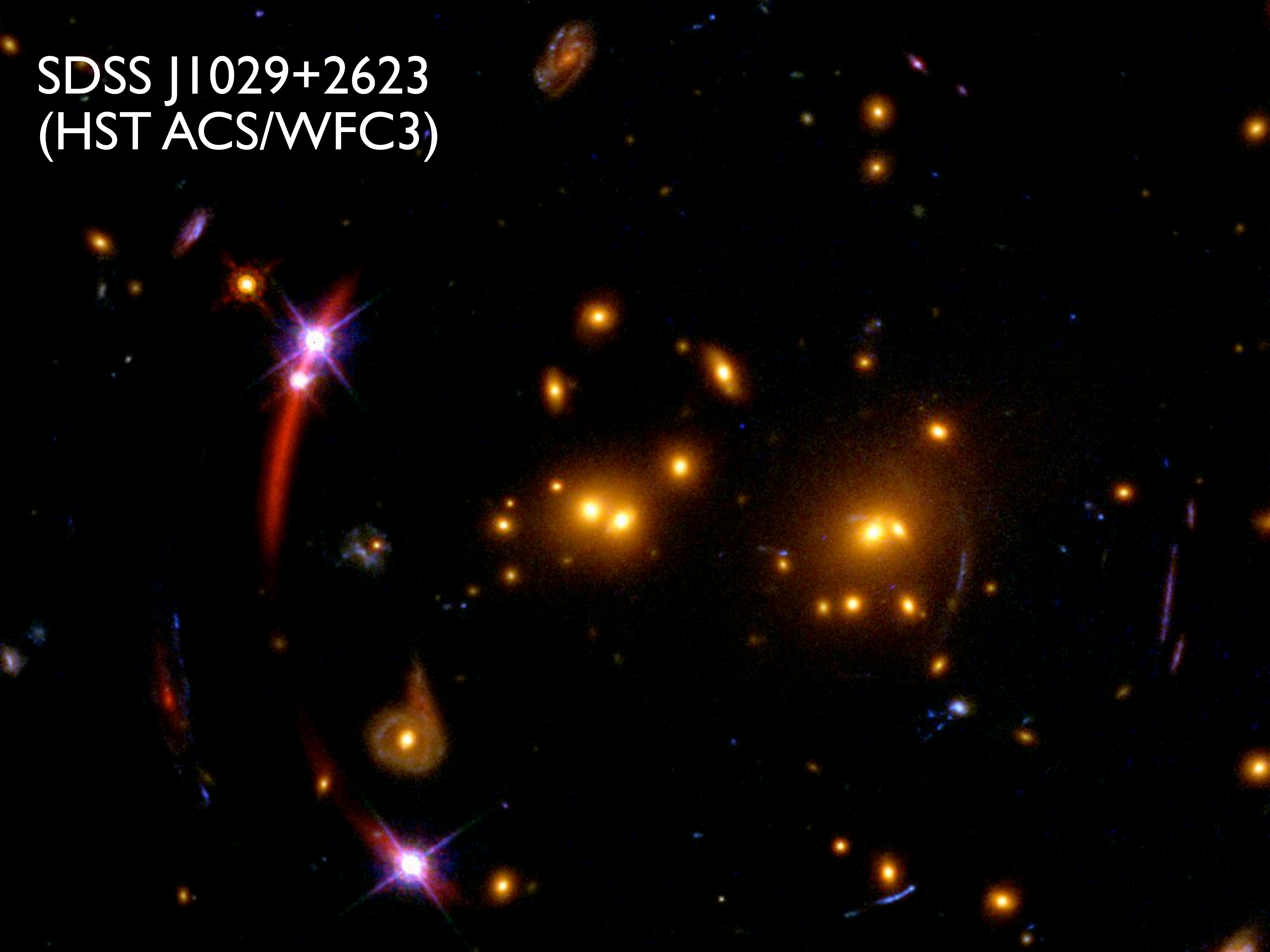


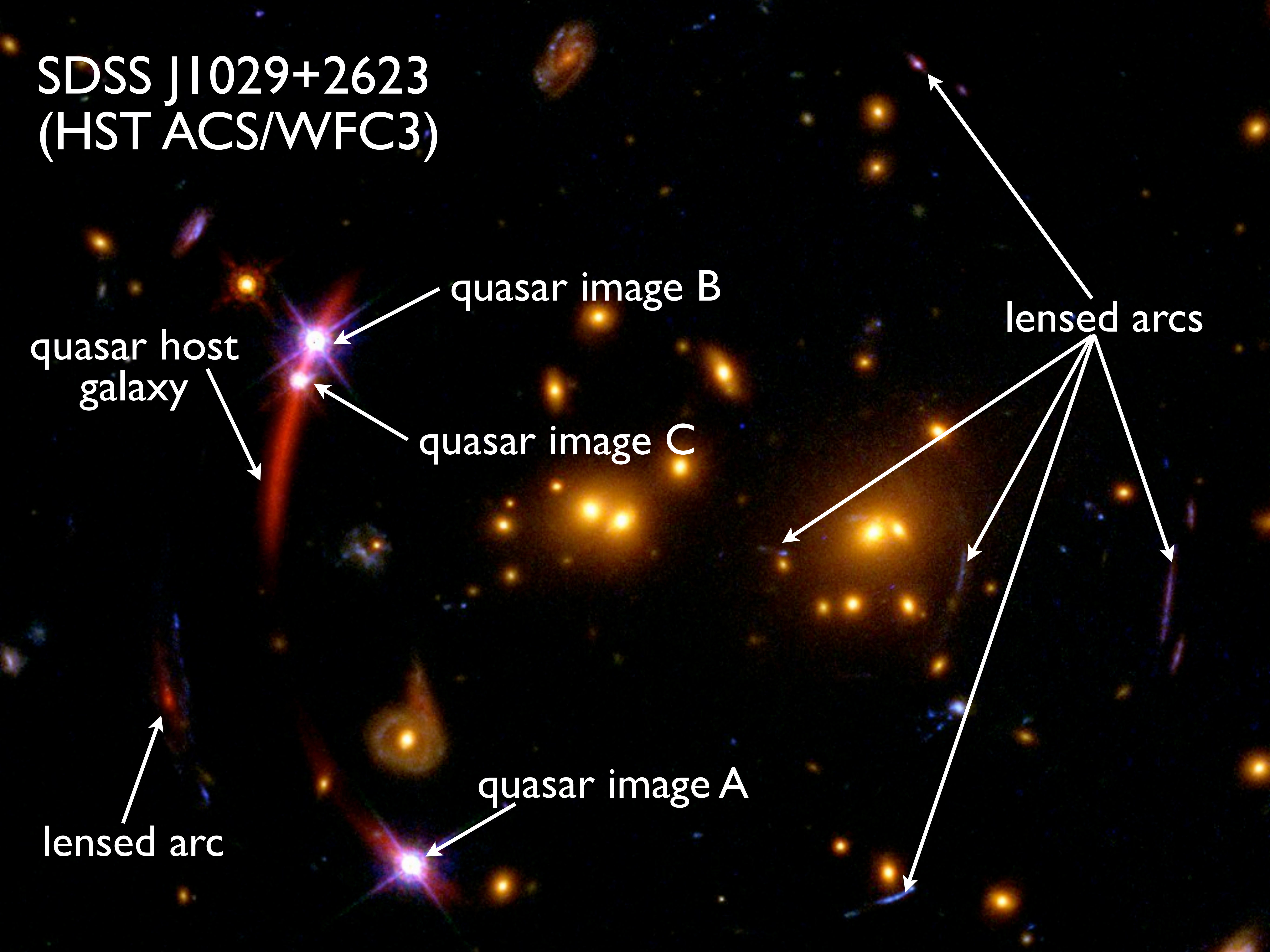
spectra taken at KPNO 2.1m

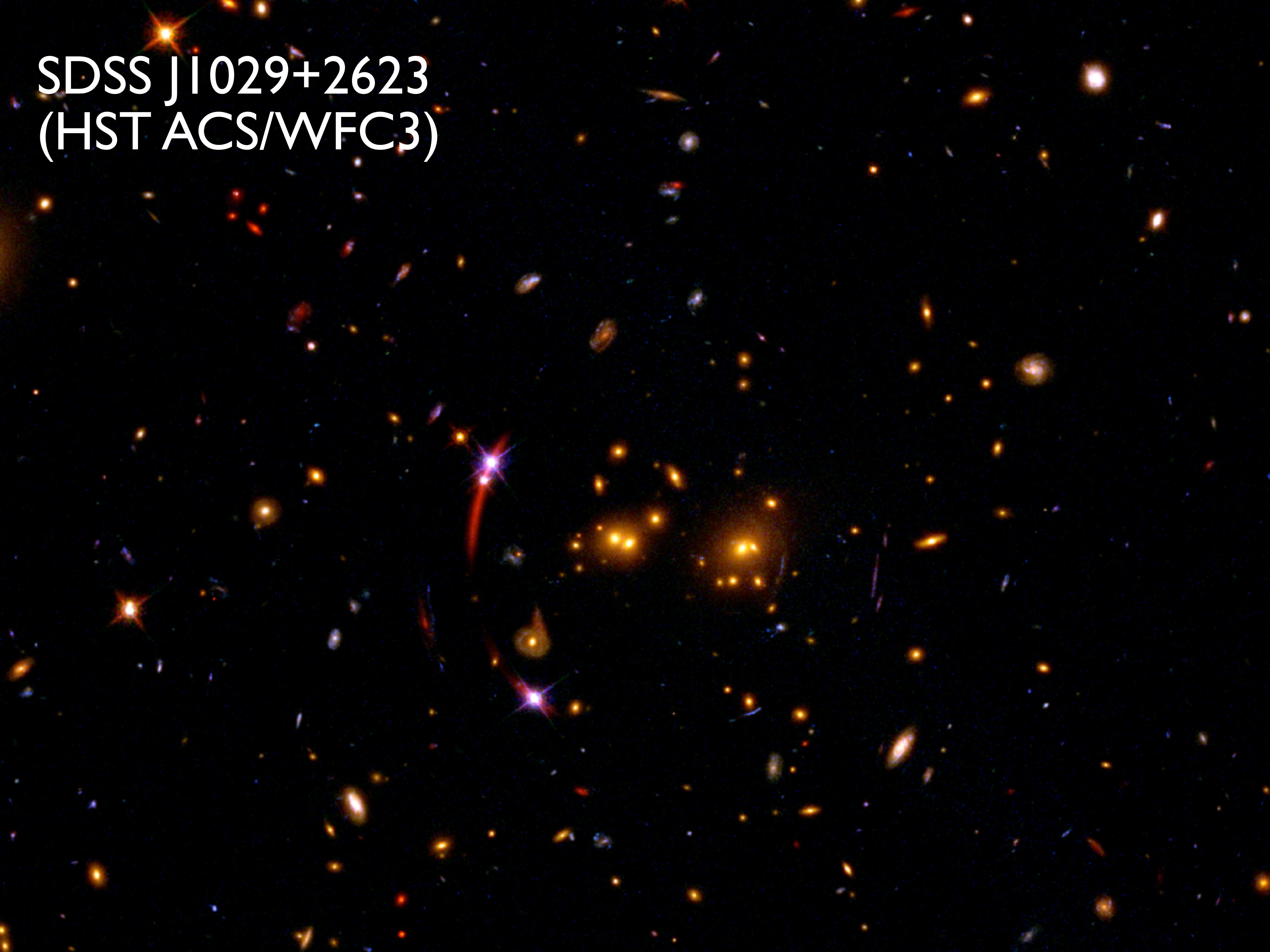
First strong lens: Q0957+561 (Walsh et al. 1979)




M. Gladders et al.


SDSS J1029+2623
(HST ACS/WFC3)


SDSS J1029+2623
(HST ACS/WFC3)
quasar image B
quasar host galaxy
quasar image C
lensed arcs
quasar image A
lensed arc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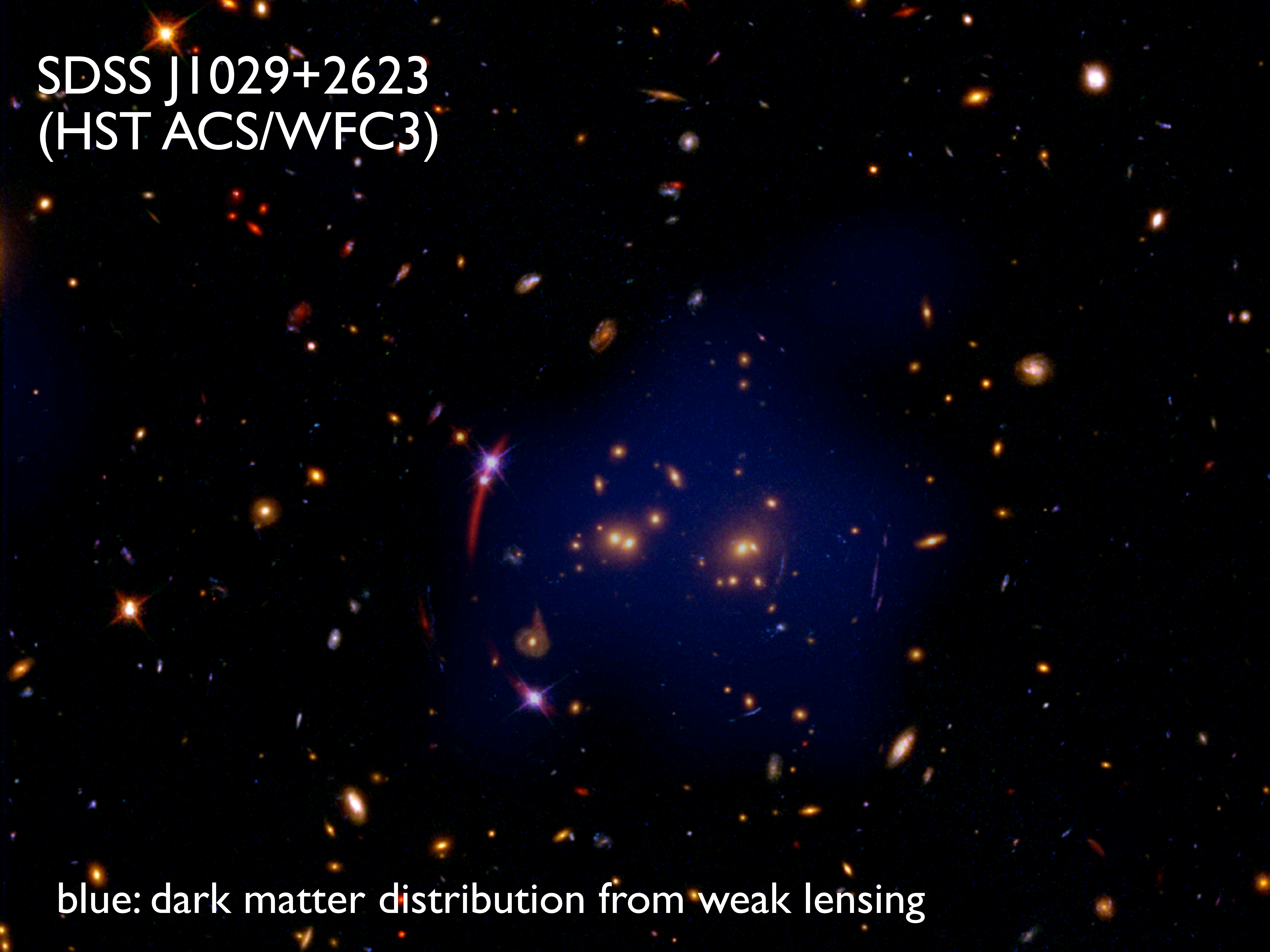

SDSS J1029+2623
(HST ACS/WFC3)
blue: dark matter distribution from weak lensing

Basics of gravitational lensing

- derivation of ‘lens equation’
- convergence, shear, magnification
- critical curves and caustics
- time delay

Lens equation

- master equation of gravitational lensing
- starting point of (almost) all lensing studies
- derived unambiguously from General Relativity

Deriving lens equation: outline (I)

- metric (ϕ: Newtonian potential)

$$ds^2 = -\left(1 + \frac{2\phi}{c^2}\right) c^2 dt^2 + a^2 \left(1 - \frac{2\phi}{c^2}\right) \gamma_{ij} dx^i dx^j$$

- geodesic equation

$$\frac{dp^\mu}{d\lambda} + \Gamma^\mu{}_{\alpha\beta} p^\alpha p^\beta = 0$$

$$p^\mu \equiv \frac{dx^\mu}{d\lambda}$$

$$n^i \equiv \frac{p^i}{\sqrt{\gamma_{ij} p^i p^j}}$$ (←direction of light propagation)

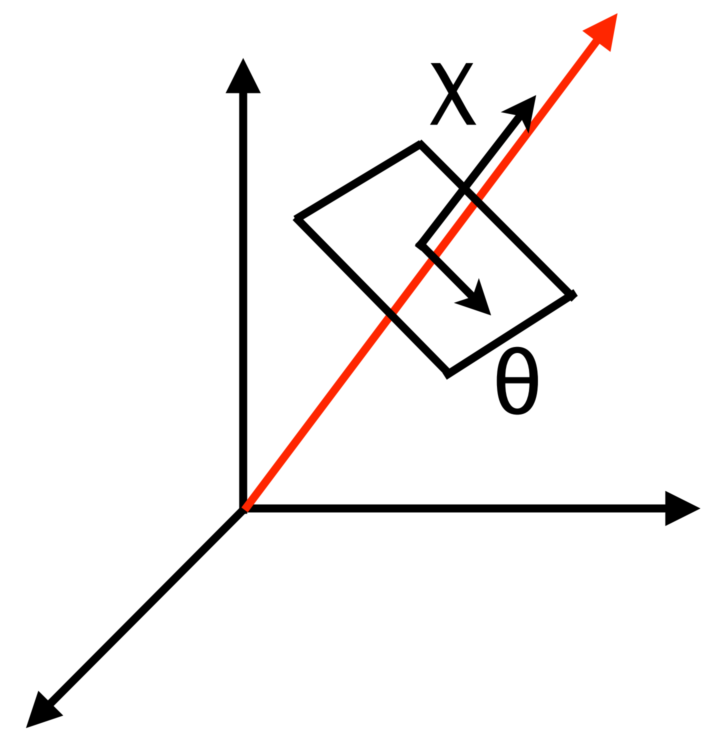
Deriving lens equation: outline (II)

- split into l.o.s. and angle coordinates

$$n^i = (\chi,\, \theta^a)$$

$$\gamma_{ij} dx^i dx^j = d\chi^2 + f_K^2(\chi)\omega_{ab} d\theta^a d\theta^b$$

$$\begin{aligned} f_K(\chi) &= \frac{1}{-K}\sinh(-K\chi) \quad (K<0) \\ &= \chi \quad (K=0) \\ &= \frac{1}{K}\sin(K\chi) \quad (K>0) \end{aligned}$$



(note: angular diameter distance $D_A = a f_K(\chi)$)

Deriving lens equation: outline (III)

- 0-th component of the geodesic equation
  → cosmological+gravitational redshifts

$$\frac{d(a^2p)}{d\lambda} = \frac{2a^3p^2}{c^3}\dot{\phi}$$

$$E = h\nu = ap\left(1 - \frac{\phi}{c^2}\right)$$

O: observer
S : source

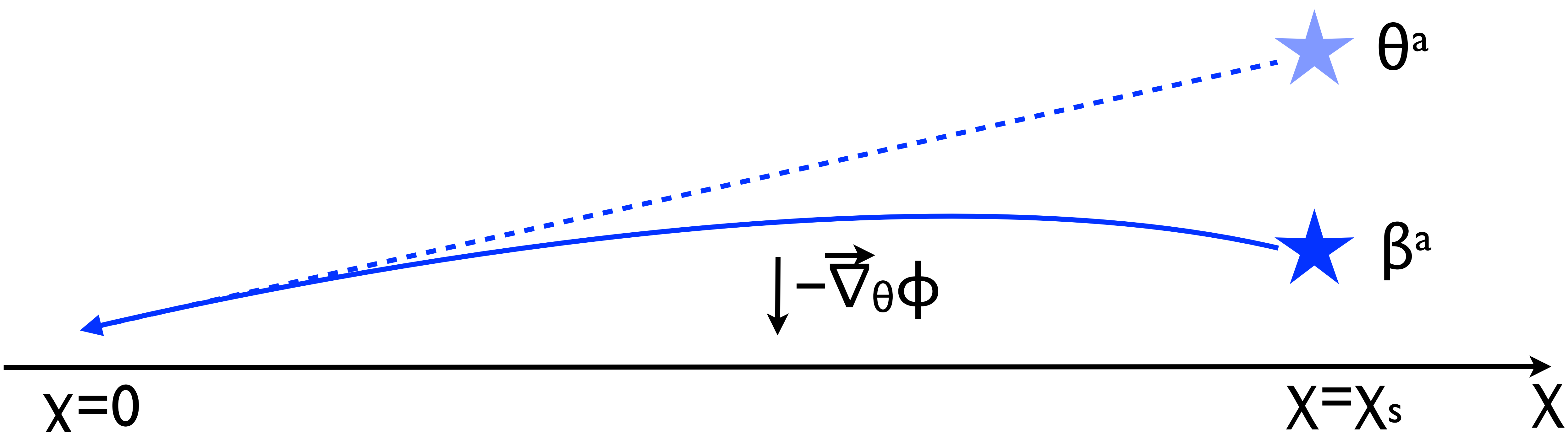
$$\rightarrow \quad 1+z = \frac{E_S}{E_O} = \underbrace{\frac{a_O}{a_S}}_{\text{cosmological redshift}}\left\{1 + \underbrace{\frac{\phi(O)-\phi(S)}{c^2}}_{\text{gravitational redshift}} - \underbrace{\frac{2}{c^2}\int_S^O \dot{\phi}\,dt}_{\text{Integrated Sachs-Wolfe}}\right\}$$

Deriving lens equation: outline (IV)

- i-th component of the geodesic equation

$$\frac{d}{d\chi}\left(f_K^2(\chi)\frac{d\theta^a}{d\chi}\right)+\frac{2}{c^2}w^{ab}\phi_{,b}=0$$

$$\to\ \beta^a=\theta^a-\frac{2}{c^2}\int_0^{\chi_s}d\chi\,\frac{f_K(\chi_s-\chi)}{f_K(\chi)f_K(\chi_s)}w^{ab}\phi_{,b}$$



Deriving lens equation: outline (V)

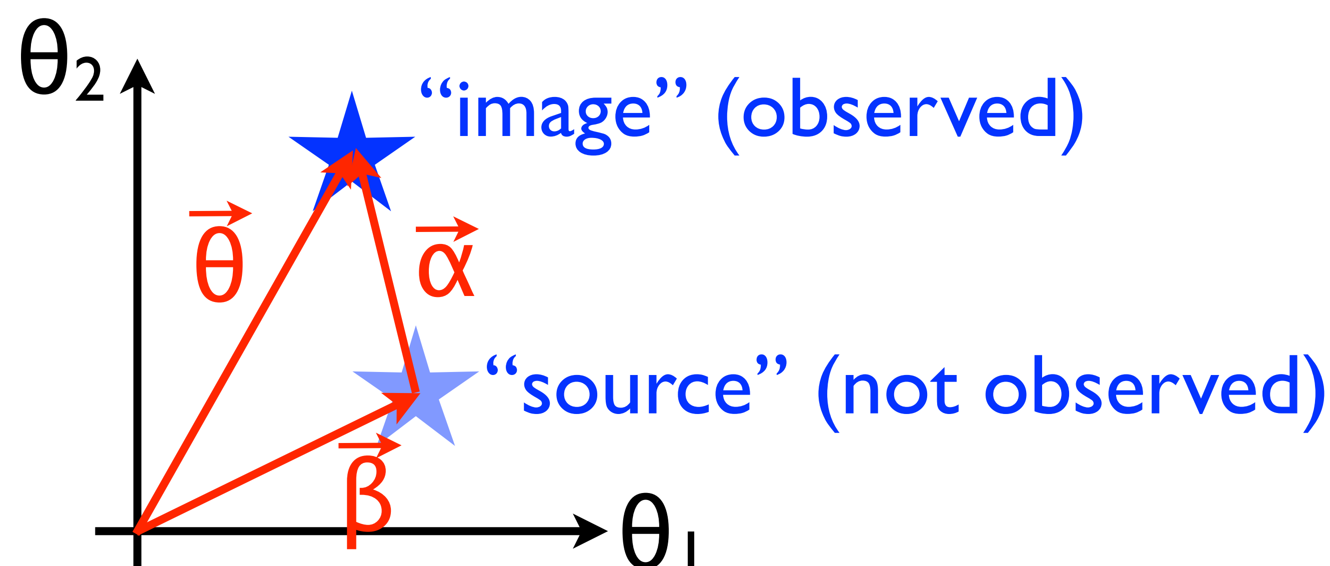
- lens equation (assuming small def. angle)

$$\boxed{\vec{\beta} = \vec{\theta} - \vec{\nabla}_\theta \psi}$$

$$\vec{\alpha}(\vec{\theta}) \equiv \vec{\nabla}_\theta \psi \quad \text{(deflection angle)}$$

$$\psi \equiv \frac{2}{c^2} \int_0^{\chi_s} d\chi \frac{f_K(\chi_s - \chi)}{f_K(\chi) f_K(\chi_s)} \phi \quad \text{(lens potential)}$$

projected coordinates on the sky



Connection to density fluctuations

- Laplacian of the lens potential

density fluctuation
δ=δρ/ρ

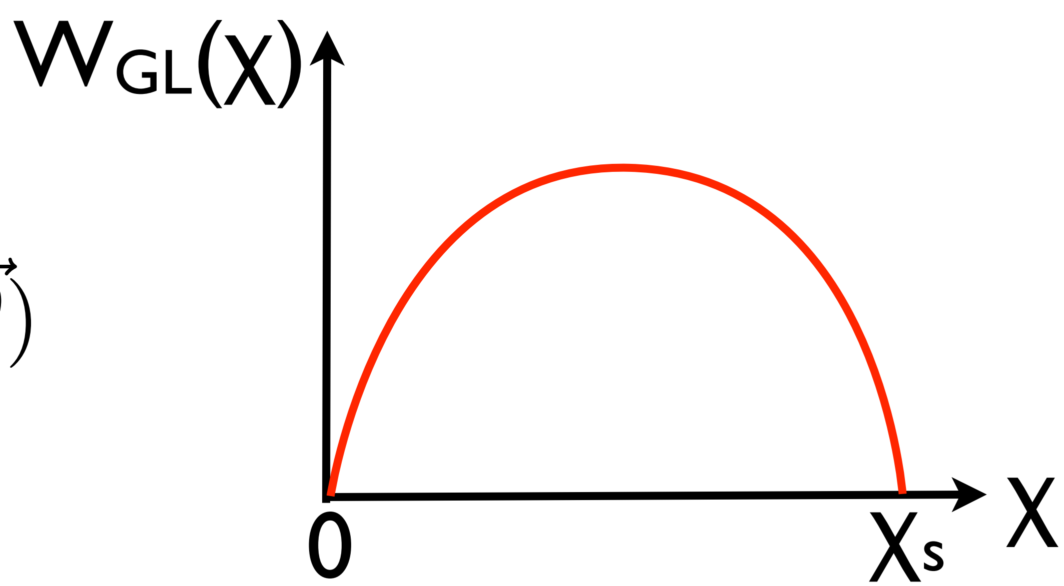
$$\vec{\nabla}^2\phi = 4\pi G a^2 \bar{\rho}\delta$$ (Poisson eq.)

$$\rightarrow \vec{\nabla}_\theta^2 \psi = 2 \times \underbrace{\frac{4\pi G}{c^2}\int_0^{\chi_s} d\chi \frac{f_K(\chi_s-\chi)}{f_K(\chi)f_K(\chi_s)} a^2 \bar{\rho}\delta(\chi,\vec{\theta})}_{\equiv\, \kappa(\vec{\theta})}$$

$\equiv \kappa(\vec{\theta})$ (convergence)

more simply,

$$\kappa(\vec{\theta}) = \int d\chi W_{\rm GL}(\chi)\delta(\chi,\vec{\theta})$$



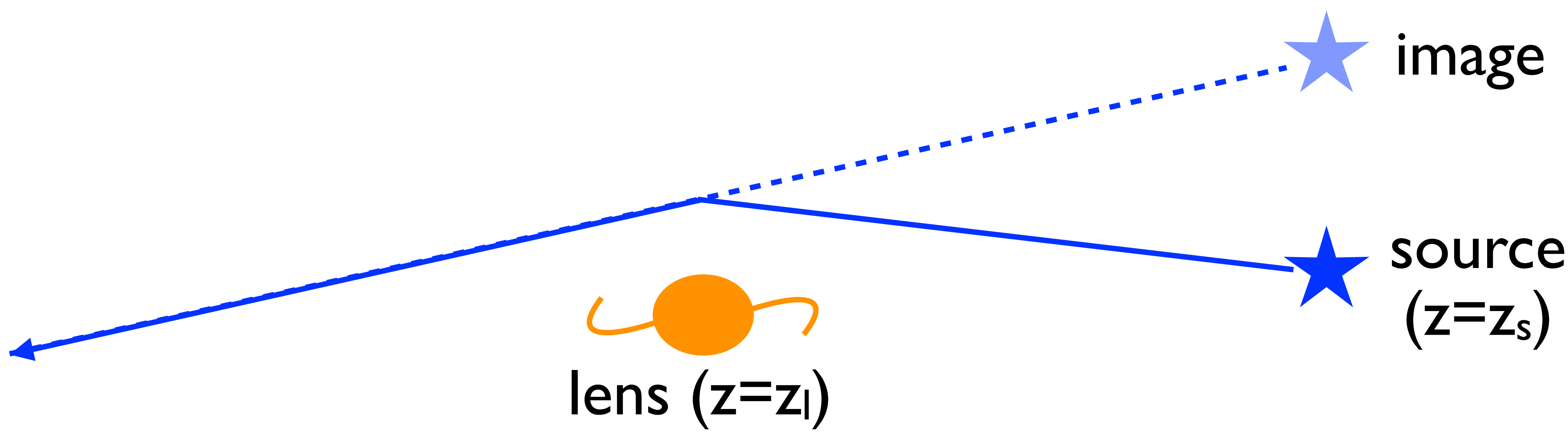
Thin lens approximation

- lens potential dominated by a single object

$$\kappa(\vec{\theta}) = \frac{\Sigma(\vec{\theta})}{\Sigma_{\rm cr}}$$

$$\Sigma(\vec{\theta}) = \int dz \delta\rho(D_A(z_l)\vec{\theta}, z)$$ (surface mass density)

$$\Sigma_{\rm cr} = \frac{c^2}{4\pi G}\frac{D_A(z_s)}{D_A(z_l, z_s) D_A(z_l)}$$ (critical surface density)



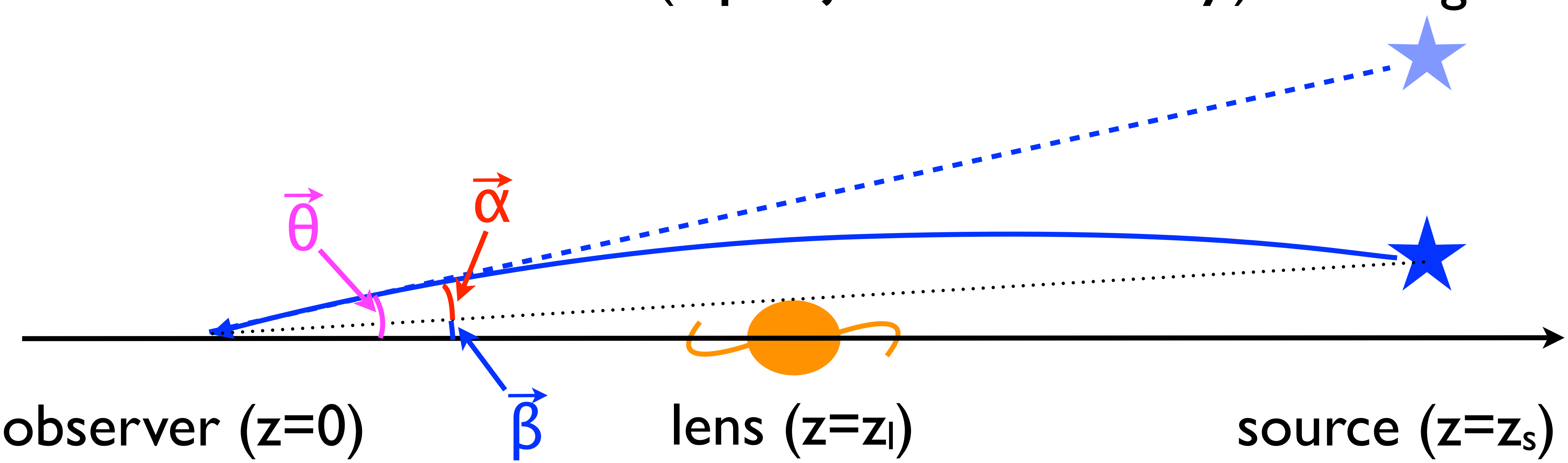
Lens equation: summary (I)

$$\boxed{\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})}$$

$$\vec{\alpha}(\vec{\theta}) = \vec{\nabla}_\theta \psi$$

$$\vec{\nabla}_\theta^2 \psi = 2\kappa(\vec{\theta})$$

$\vec{\alpha}$: deflection angle
ψ : lens potential
κ : convergence
(=projected density)



Lens equation: summary (II)

- using Green's function

$$\psi(\vec{\theta}) = \frac{1}{\pi}\int d\vec{\theta}'\,\kappa(\vec{\theta}')\ln\left|\vec{\theta}-\vec{\theta}'\right|$$

$$\vec{\alpha}(\vec{\theta}) = \frac{1}{\pi}\int d\vec{\theta}'\,\kappa(\vec{\theta}')\frac{\vec{\theta}-\vec{\theta}'}{\left|\vec{\theta}-\vec{\theta}'\right|^2}$$

mass distribution

$\downarrow$ project l.o.s

convergence κ

$\downarrow$ Green's function

lens potential ψ

$\downarrow$ derivatives

deflection angle $\vec{\alpha}$

.....

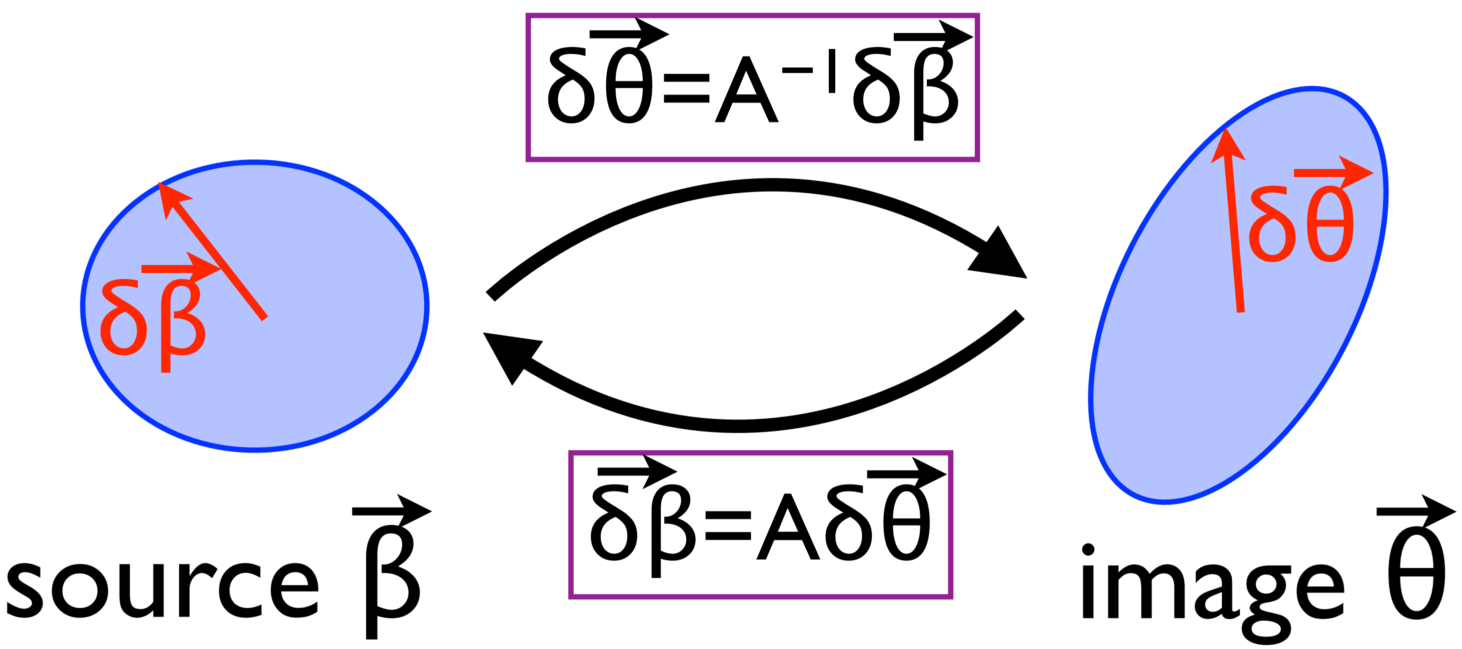
Properties of images

- lensed images are deformed by lensing

$$A \equiv \frac{\partial \vec{\beta}}{\partial \vec{\theta}} = \begin{pmatrix} 1-\psi_{11} & -\psi_{12} \\ -\psi_{12} & 1-\psi_{22} \end{pmatrix}$$

$\psi_{11} = \partial^2\psi/\partial\theta_1^2$
etc.

$$= \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$



A : *de*-lensing
A^{-1}: lensing

Convergence and shear

$$A = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$

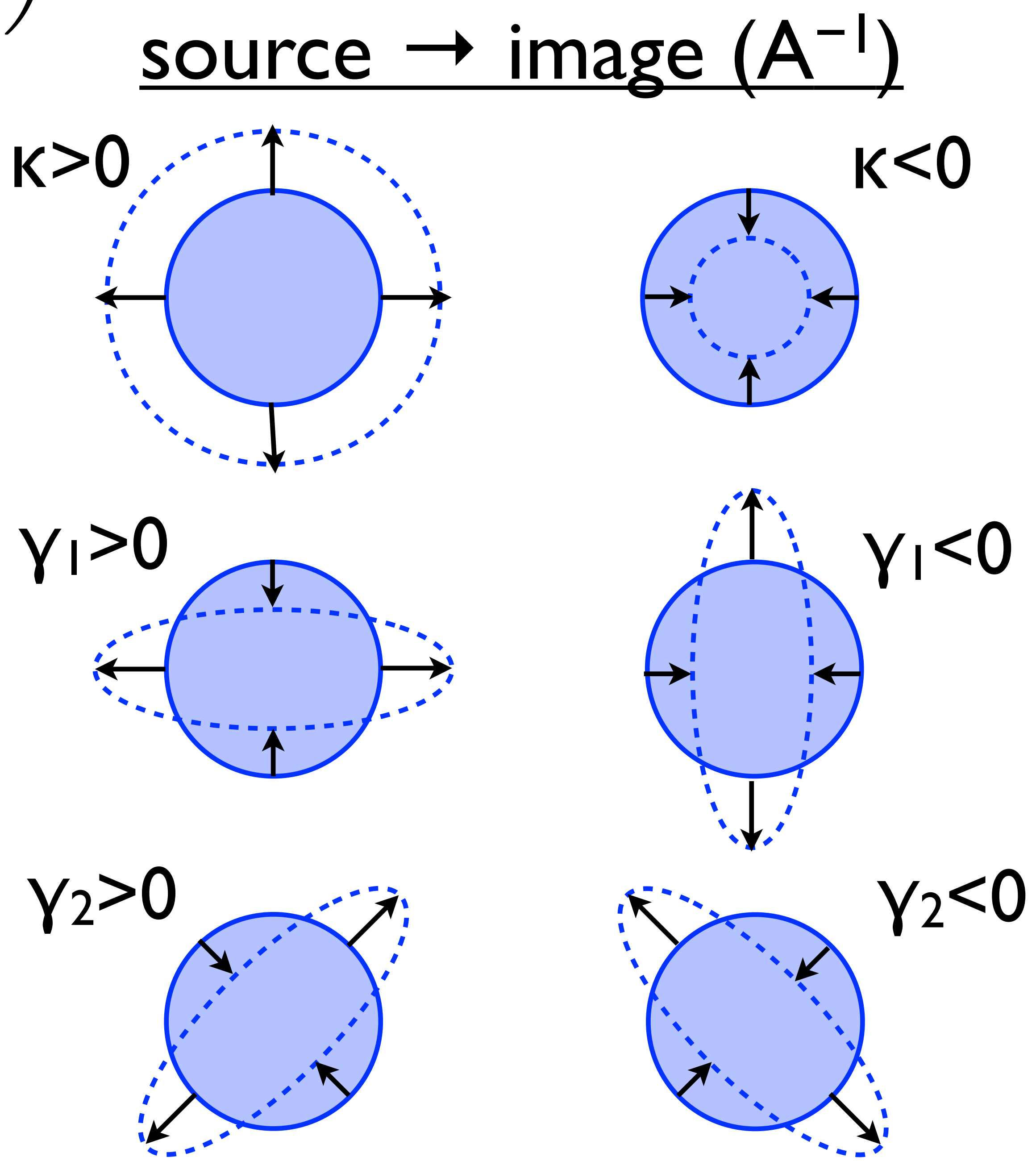
convergence

$$\kappa \equiv \frac{1}{2}(\psi_{11}+\psi_{22})$$

shear

$$\gamma_1 \equiv \frac{1}{2}(\psi_{11}-\psi_{22})$$

$$\gamma_2 \equiv \psi_{12}$$



Magnification

- lensing conserves surface brightness
  →magnification ∝ area

magnification μ ($L_{obs}=\mu L_{ori}$)

$$\mu \equiv (\det A)^{-1} = \frac{1}{(1-\kappa)^2 - |\gamma|^2}$$

$$|\gamma| \equiv \sqrt{\gamma_1^2 + \gamma_2^2}$$

Critical curves and caustics

- critical curves are defined by

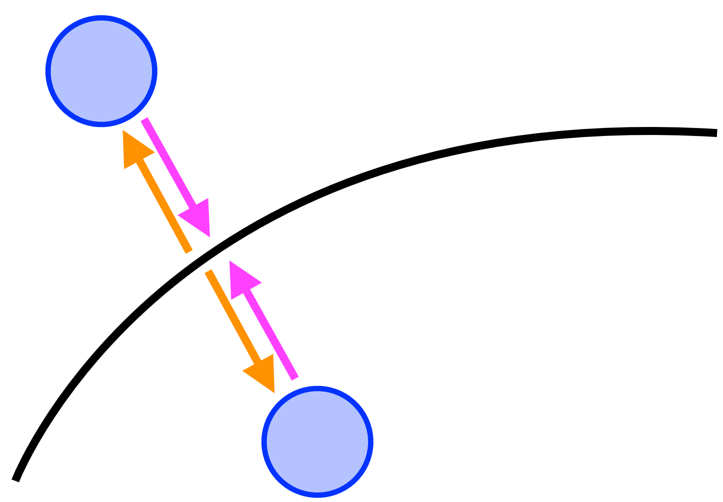
$$\det A(\vec{\theta}_c) = 0 \quad \text{(image plane)}$$

[magnification μ diverges on critical curves]

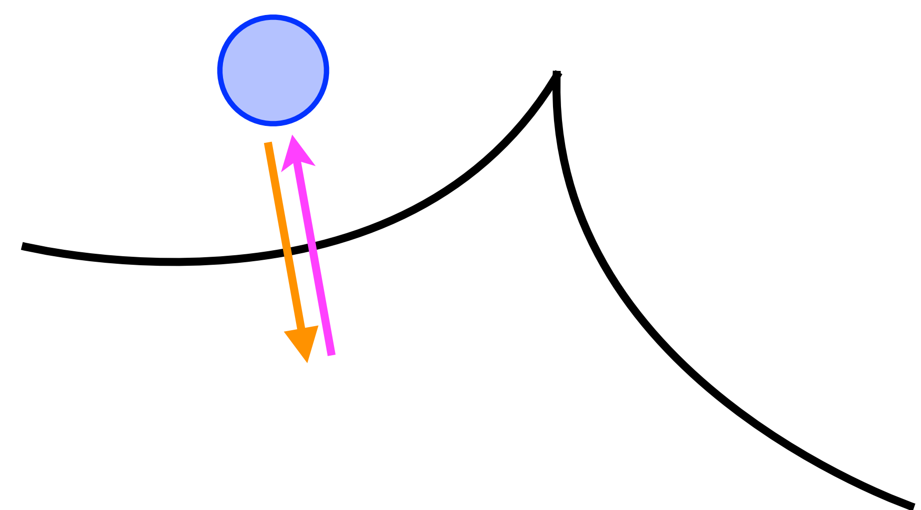
- corresponding curves in the source plane (caustics) are

$$\vec{\beta}_c = \vec{\beta}(\vec{\theta}_c) \qquad \text{(source plane)}$$

Critical curves and multiple images



critical curve
(image plane)



caustic
(source plane)

pair of images appear/disappear at critical curves

simulated by *glafic*

Example: point source (quasar)

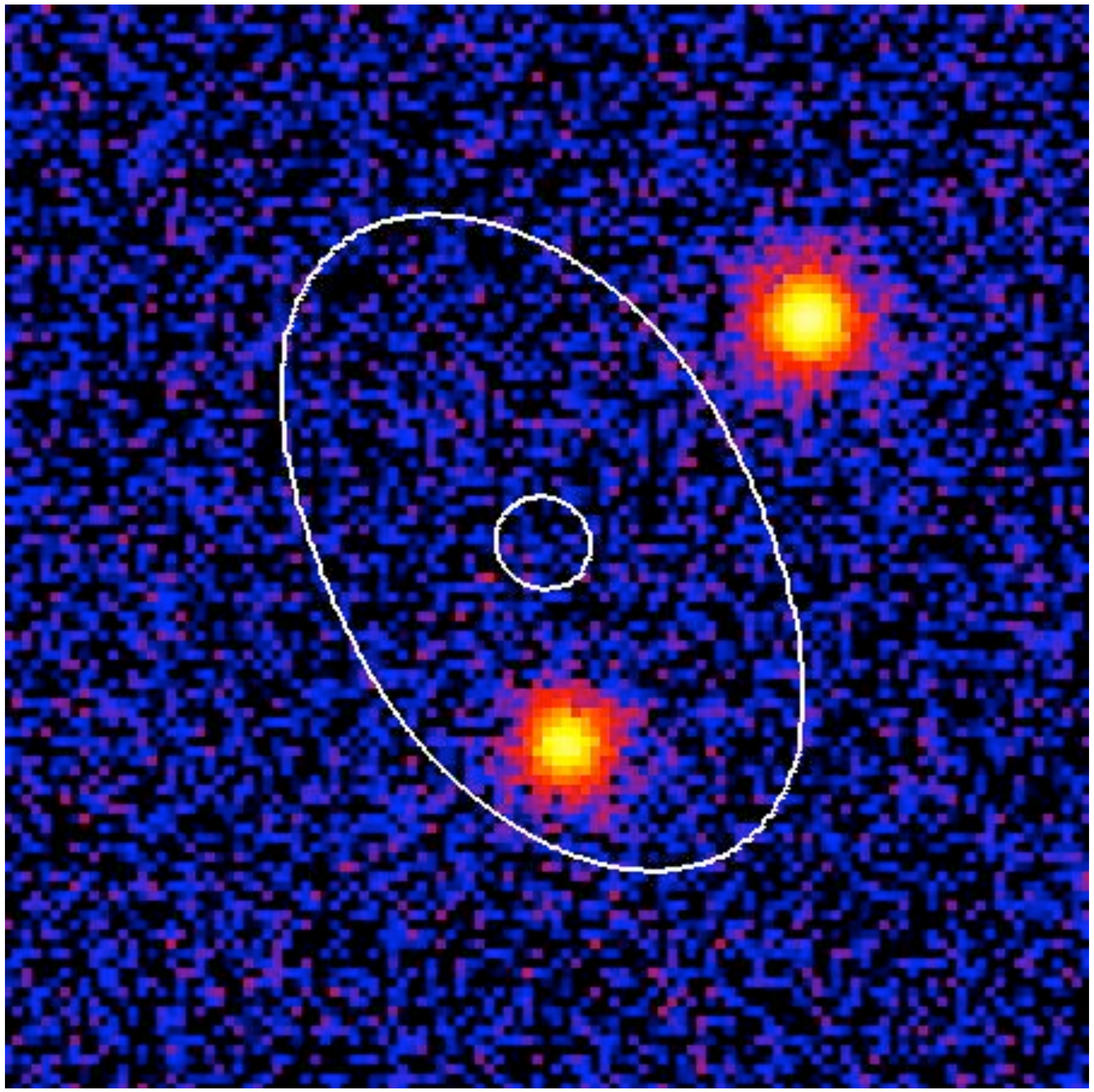
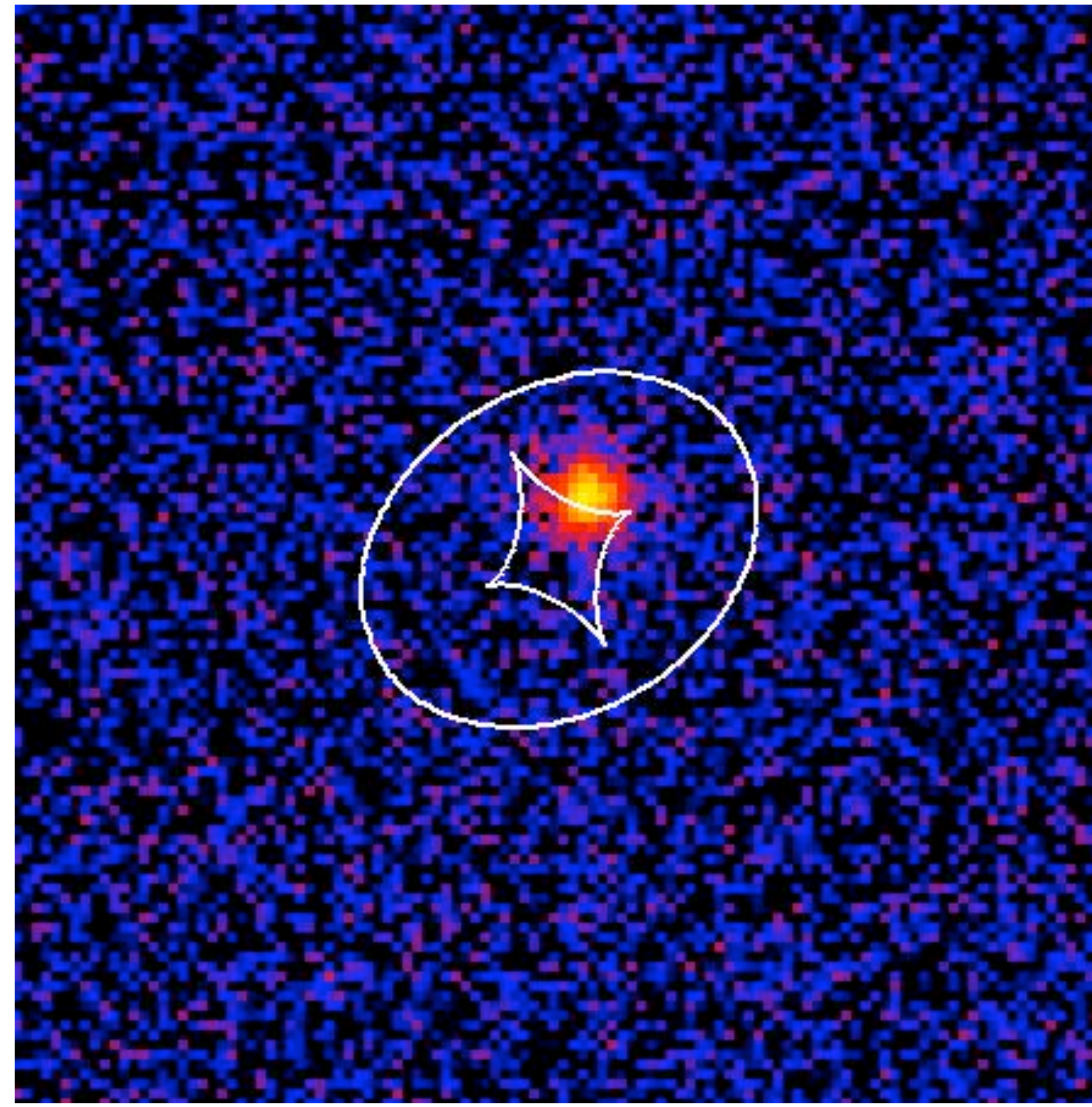


image plane
(critical curves)



source plane
(caustics)

simulated by *glafic*

Example: point source (quasar)

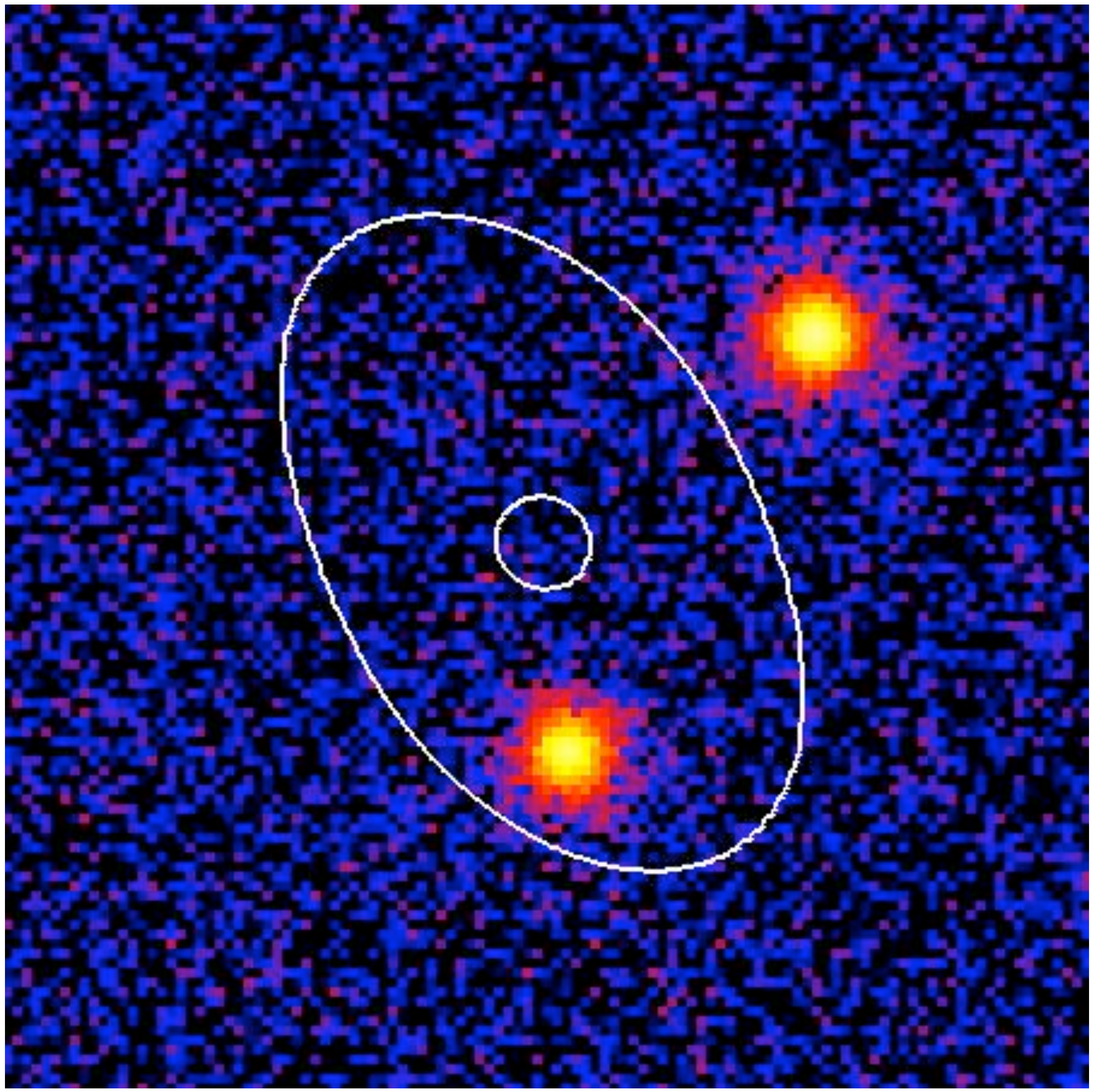
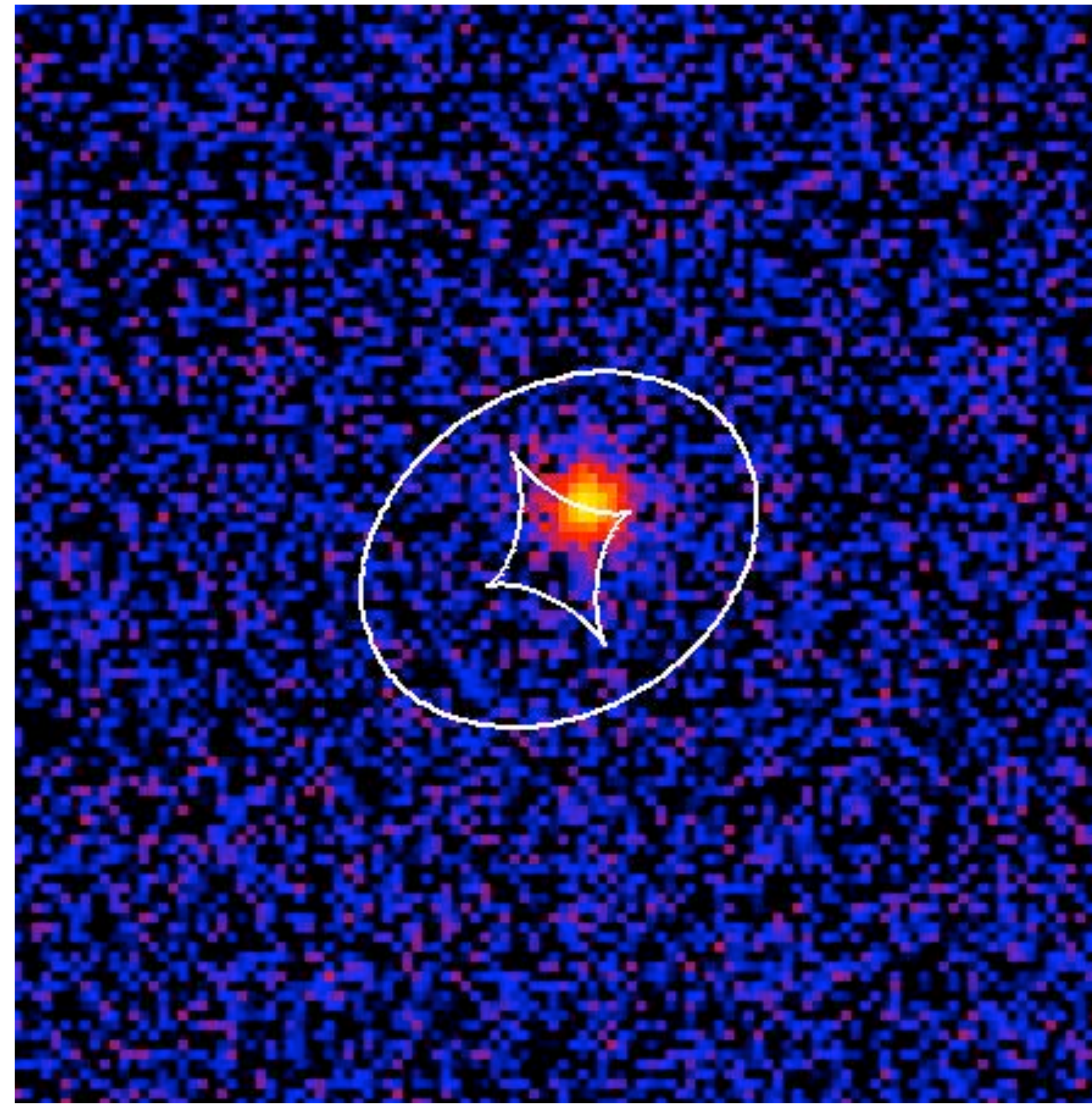


image plane
(critical curves)



source plane
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Example: point source (quasar)

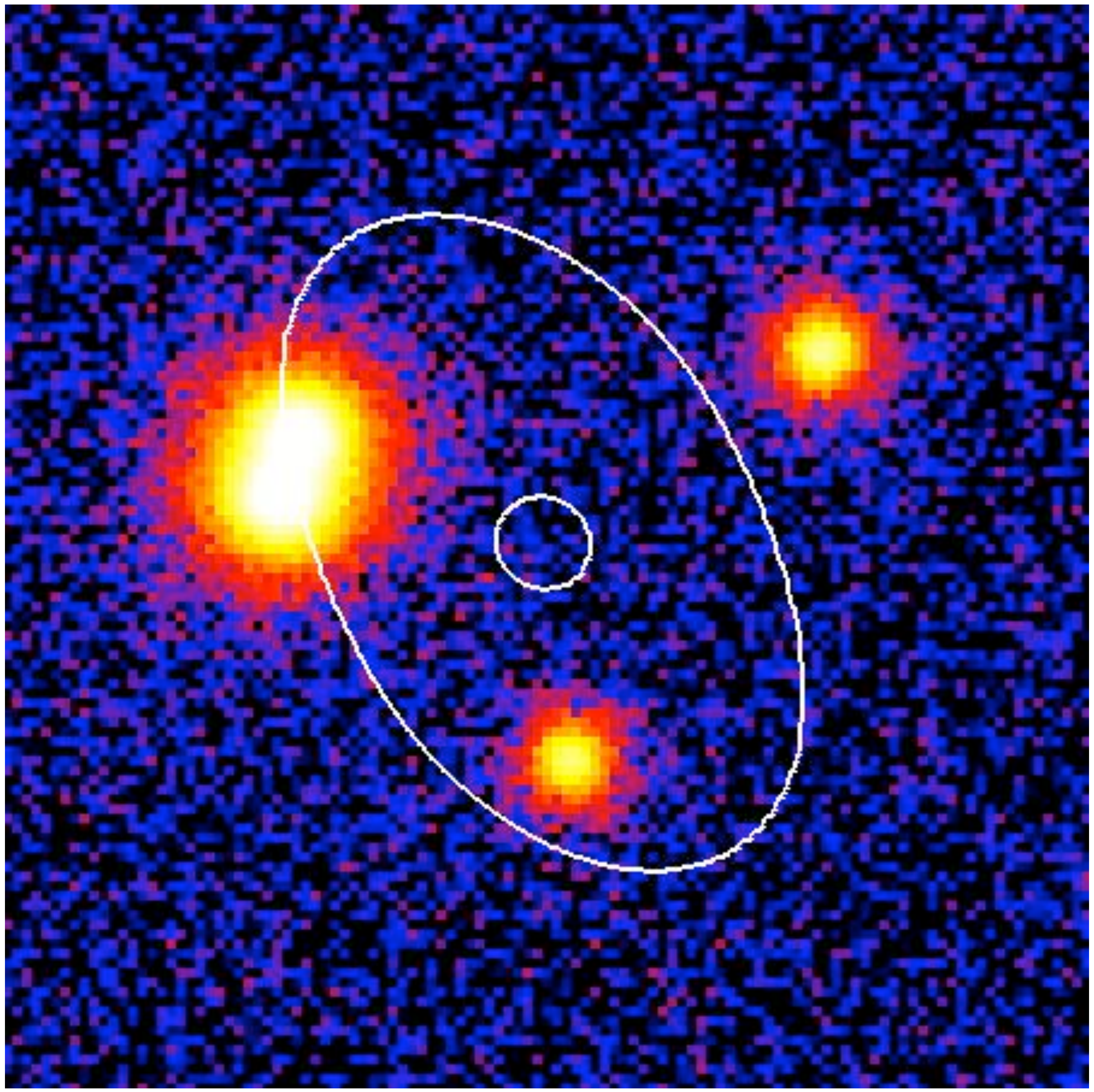
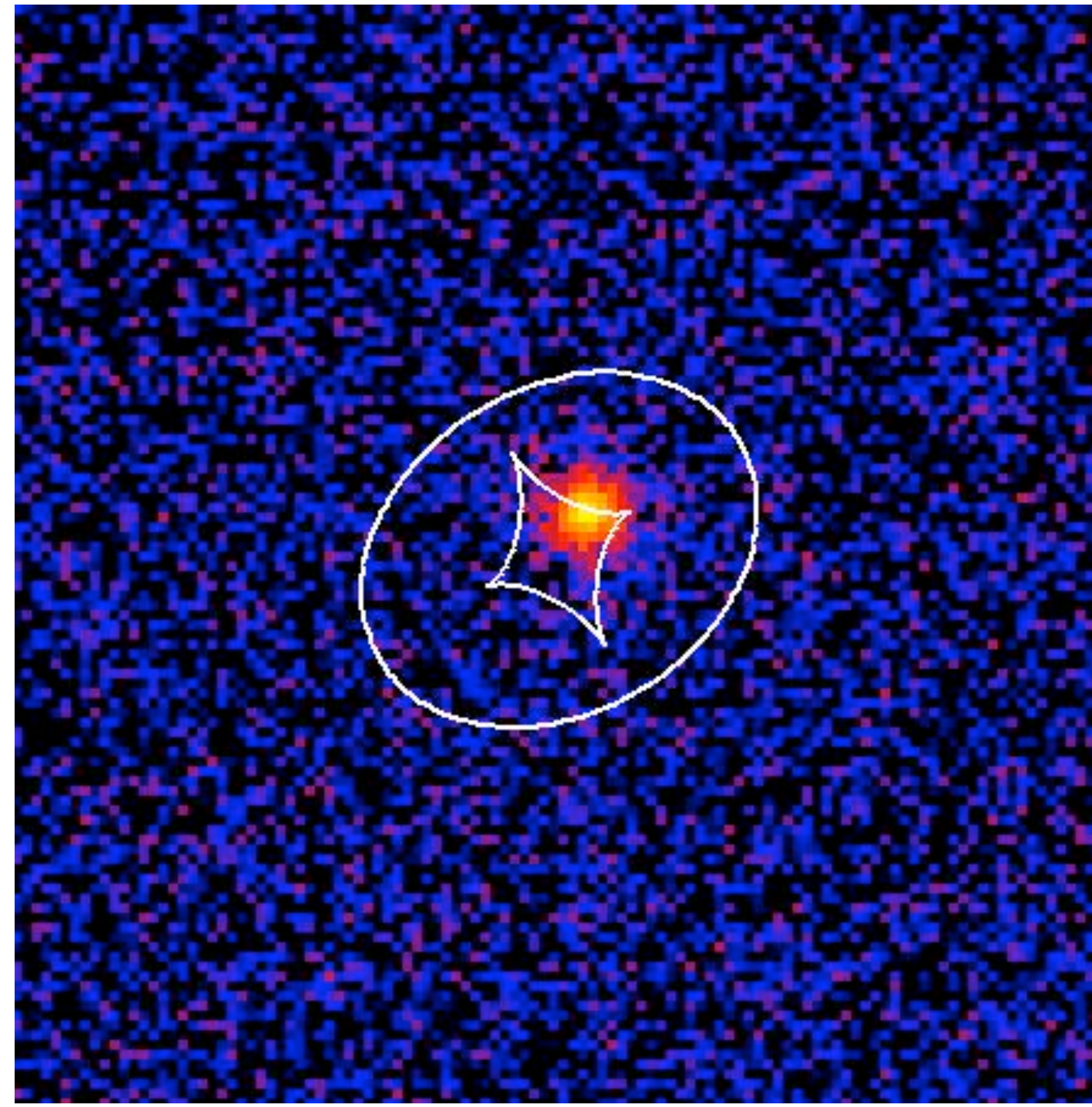


image plane
(critical curves)



source plane
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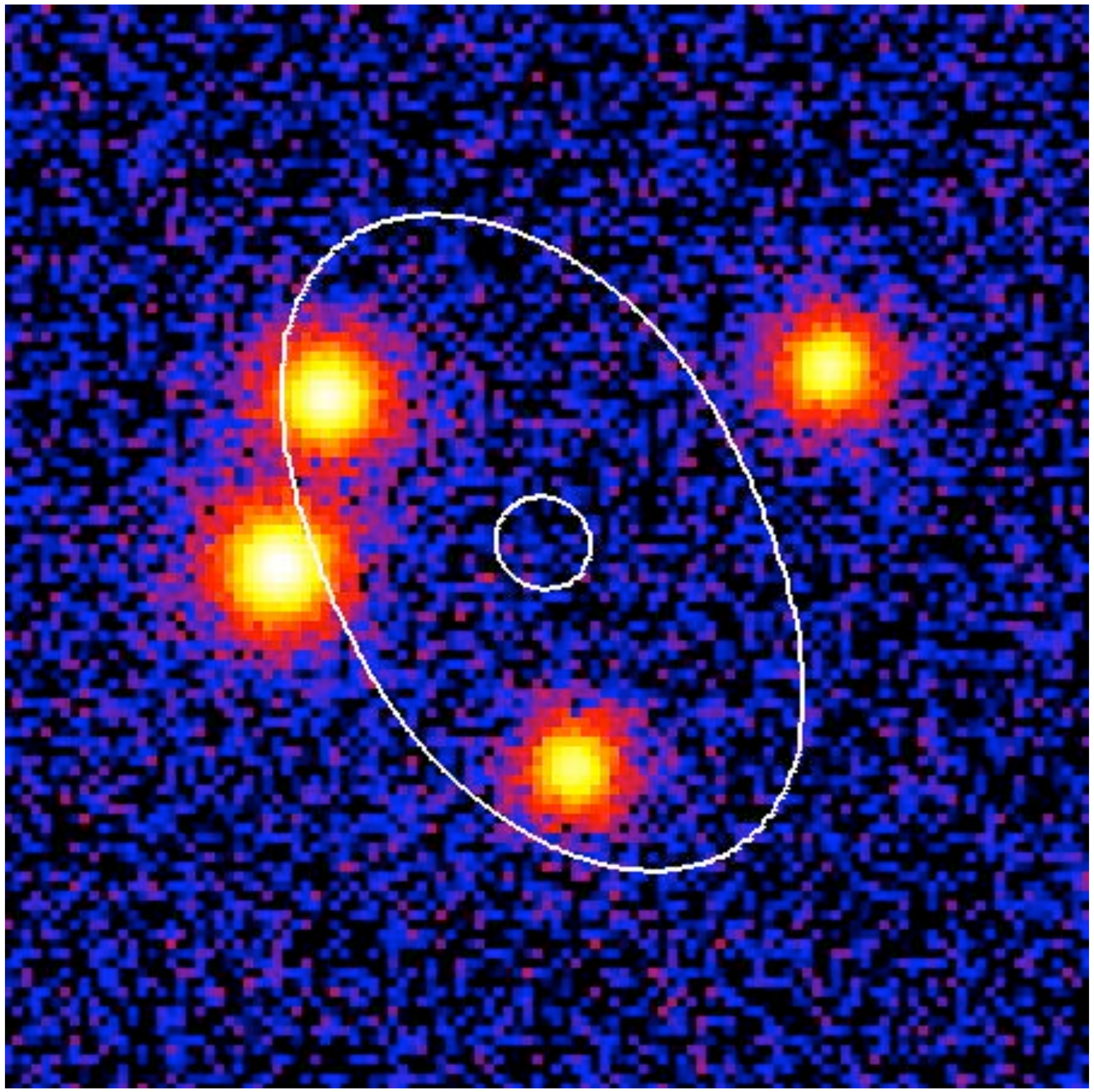
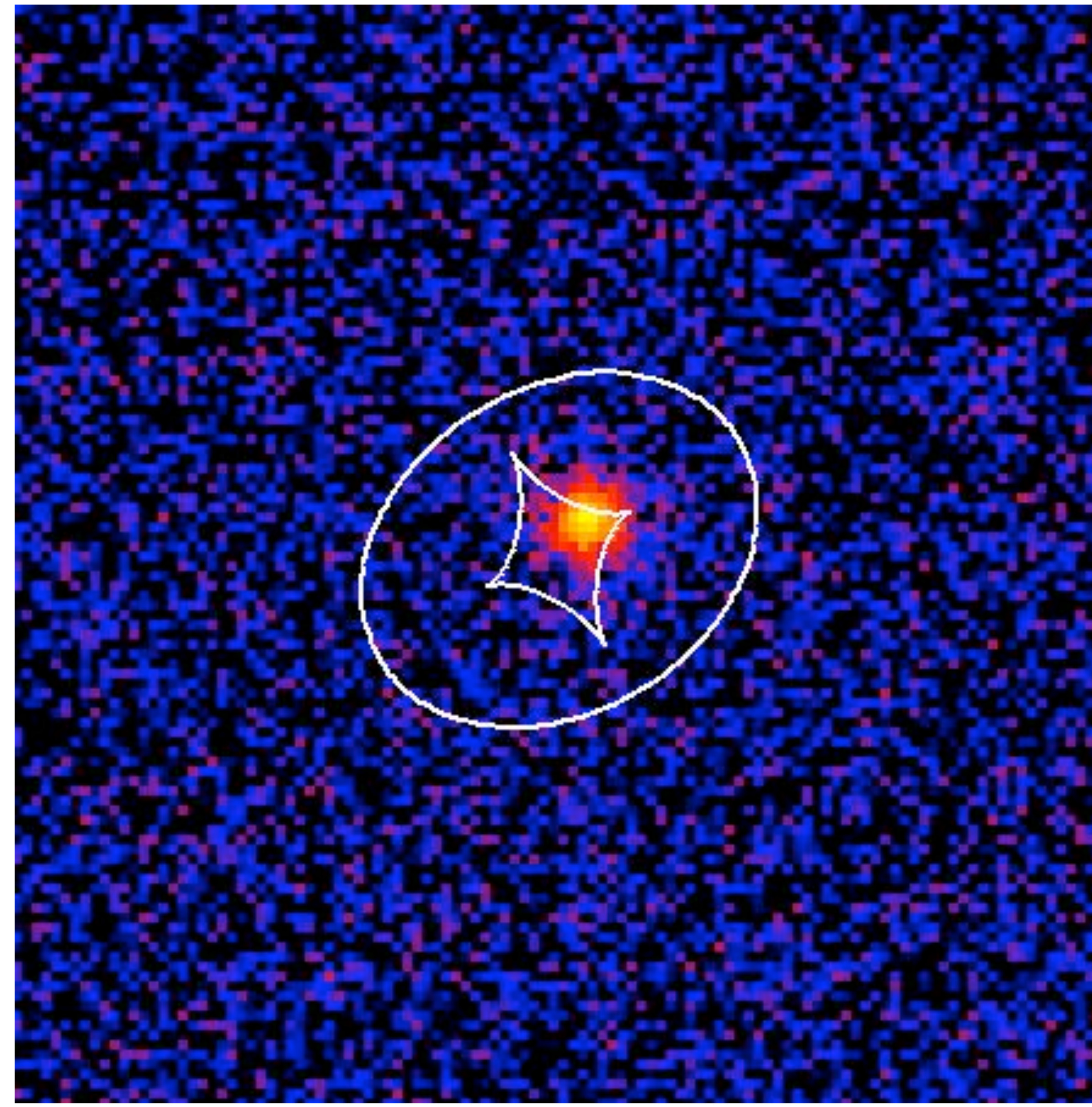


image plane
(critical curves)



source plane
(caustics)

simulated by *glafic*

Example: point source (quasar)

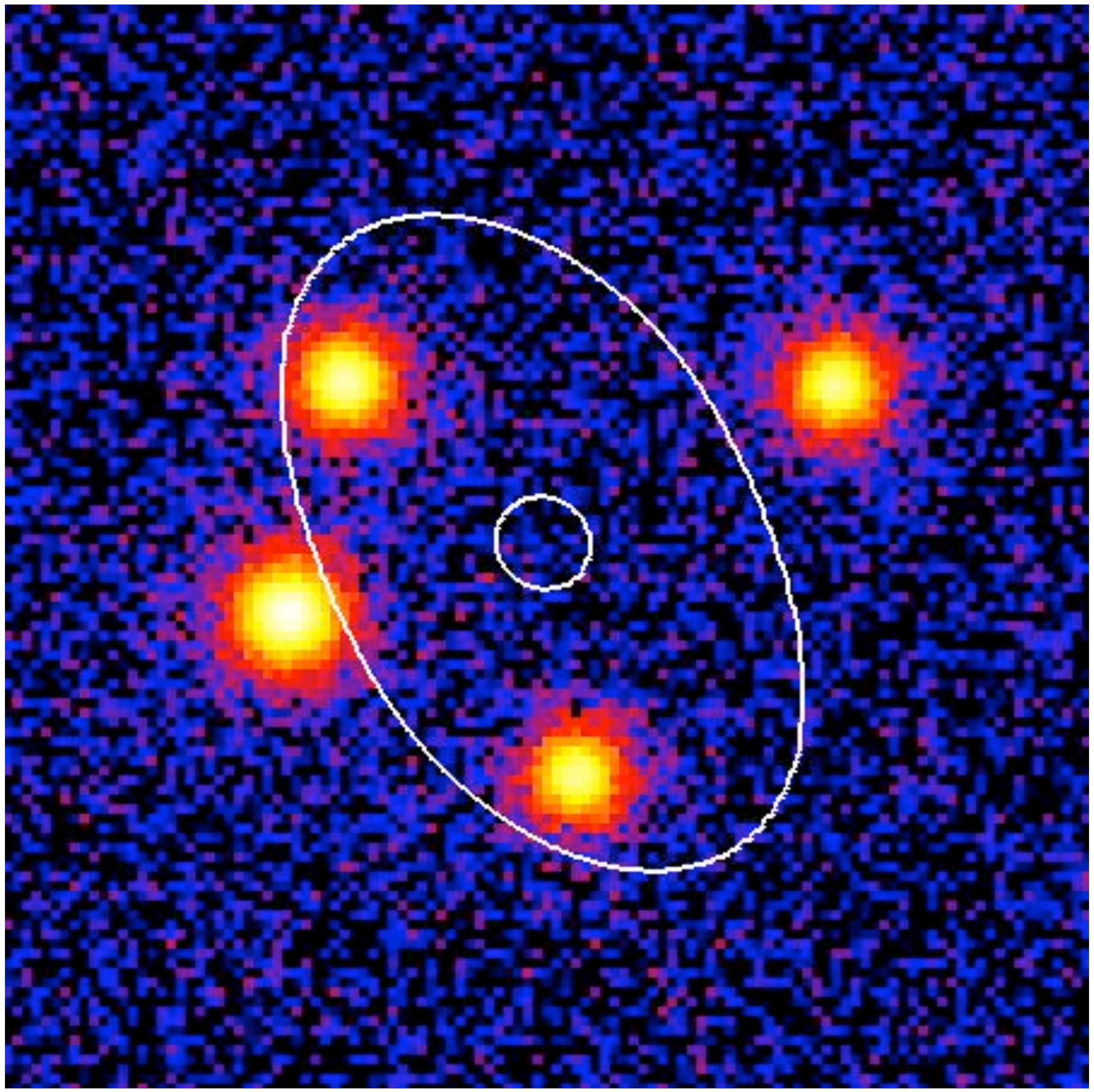
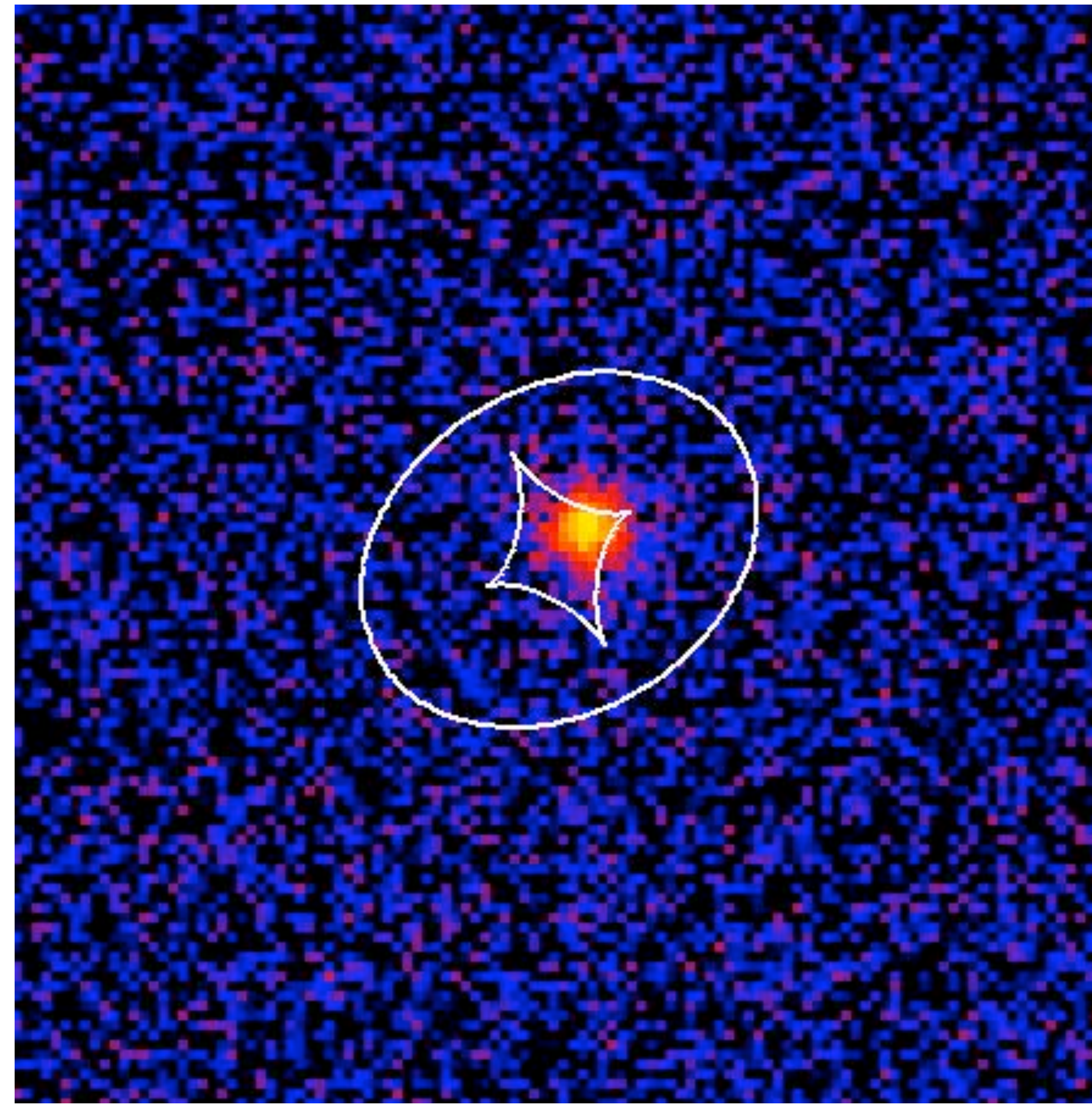


image plane
(critical curves)



source plane
(caustics)

simulated by *glafic*

Example: extended source (galaxy)

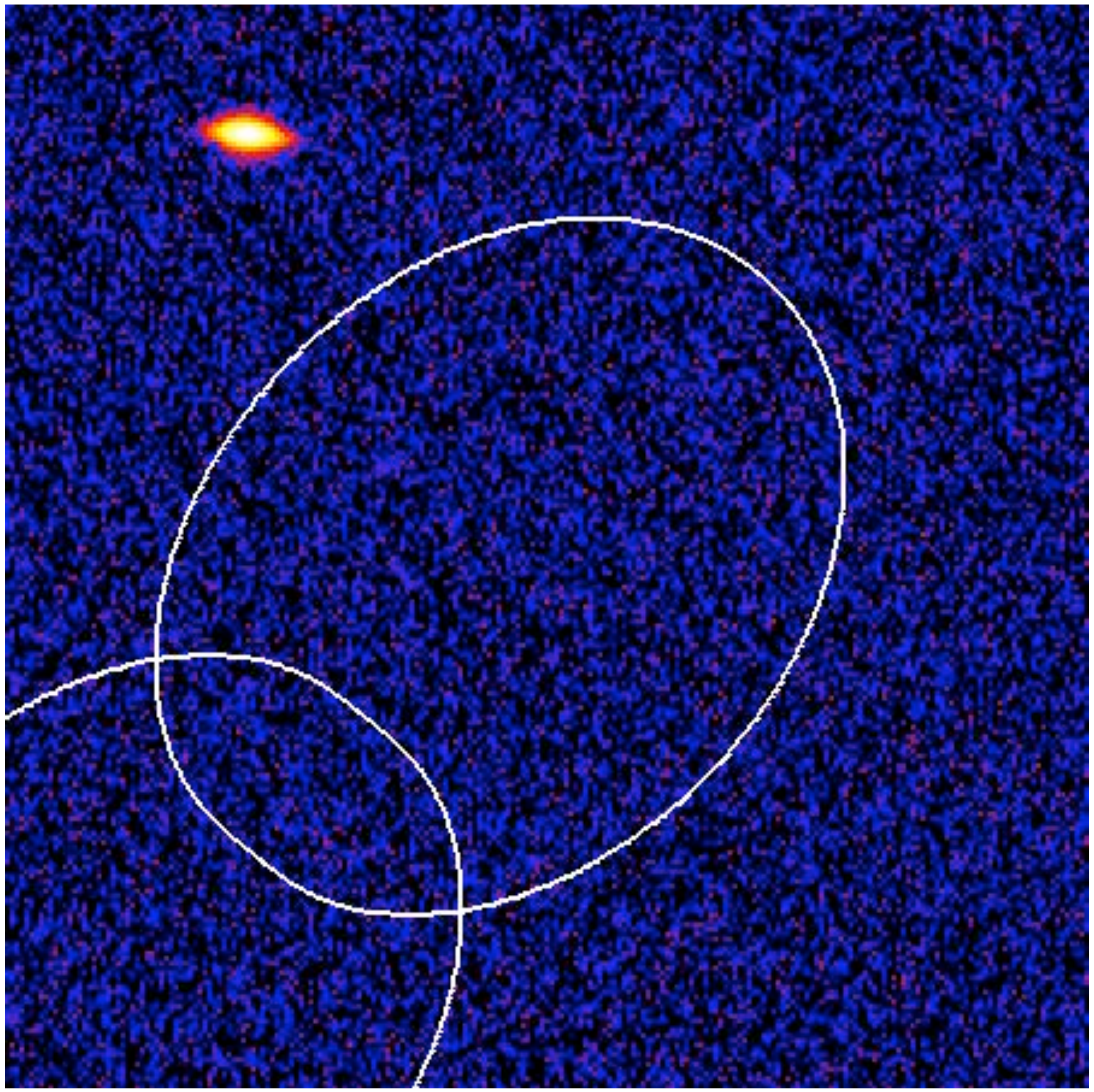
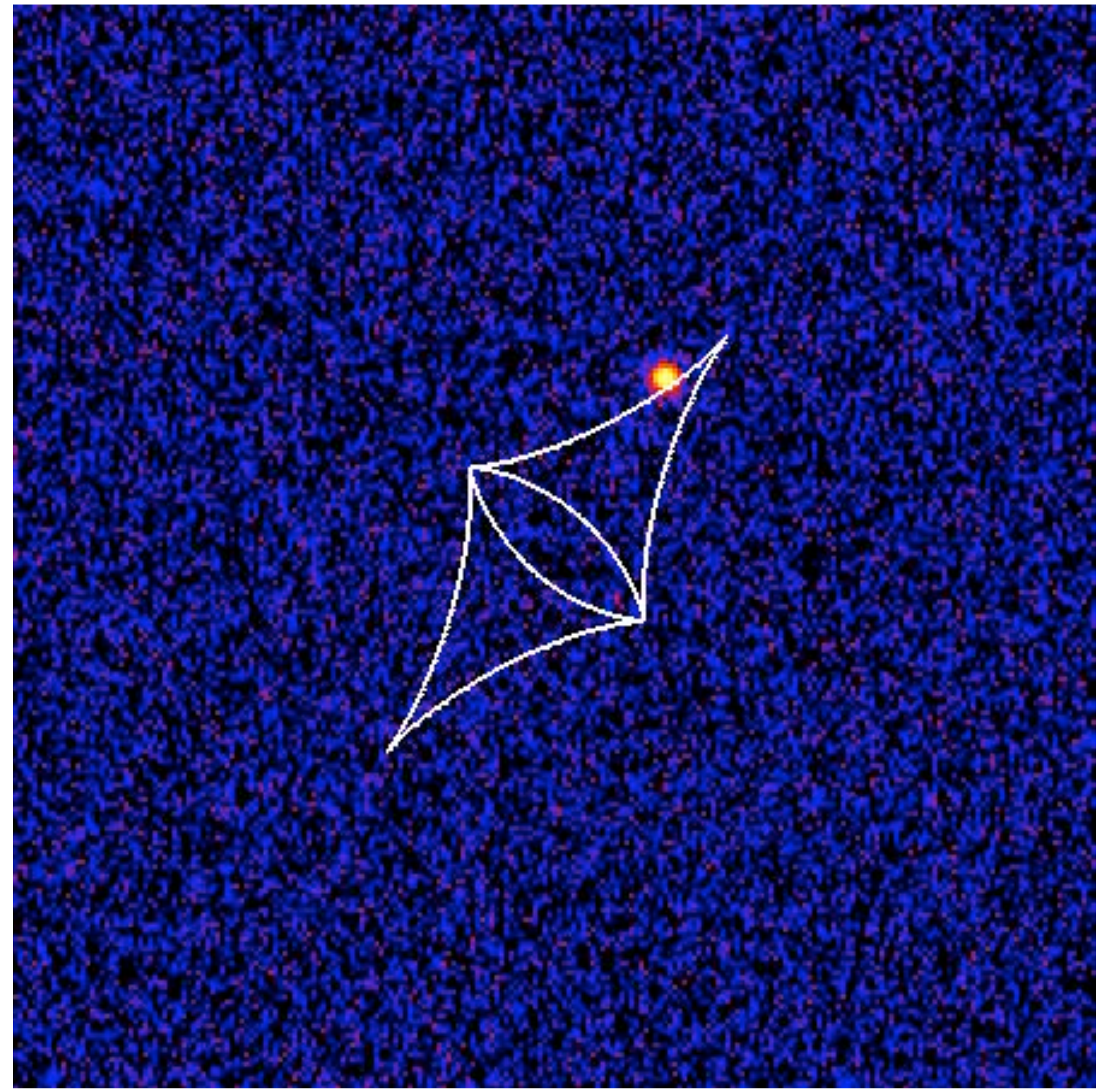


image plane
(critical curves)



source plane
(caustics)

simulated by *glafic*

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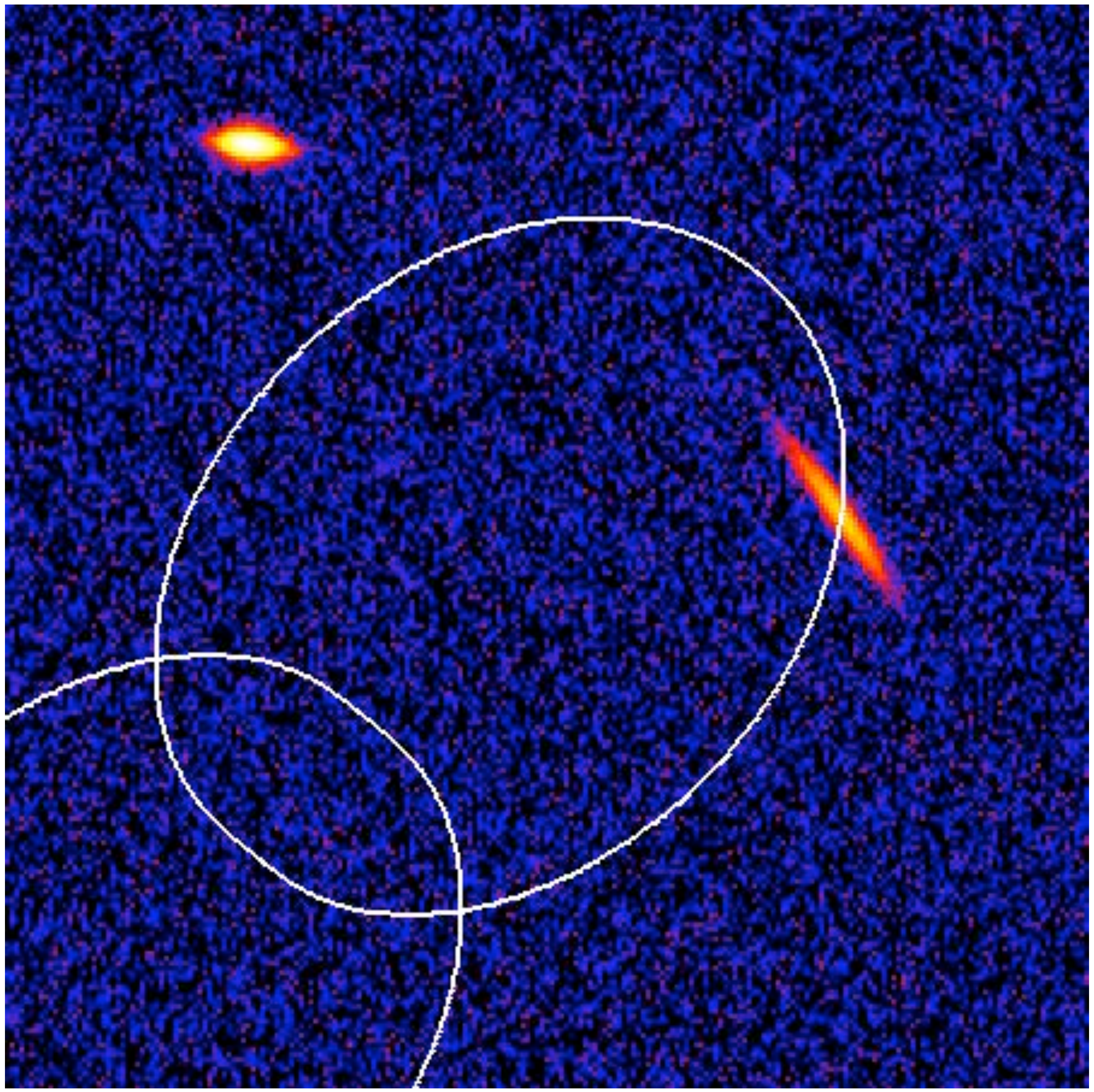
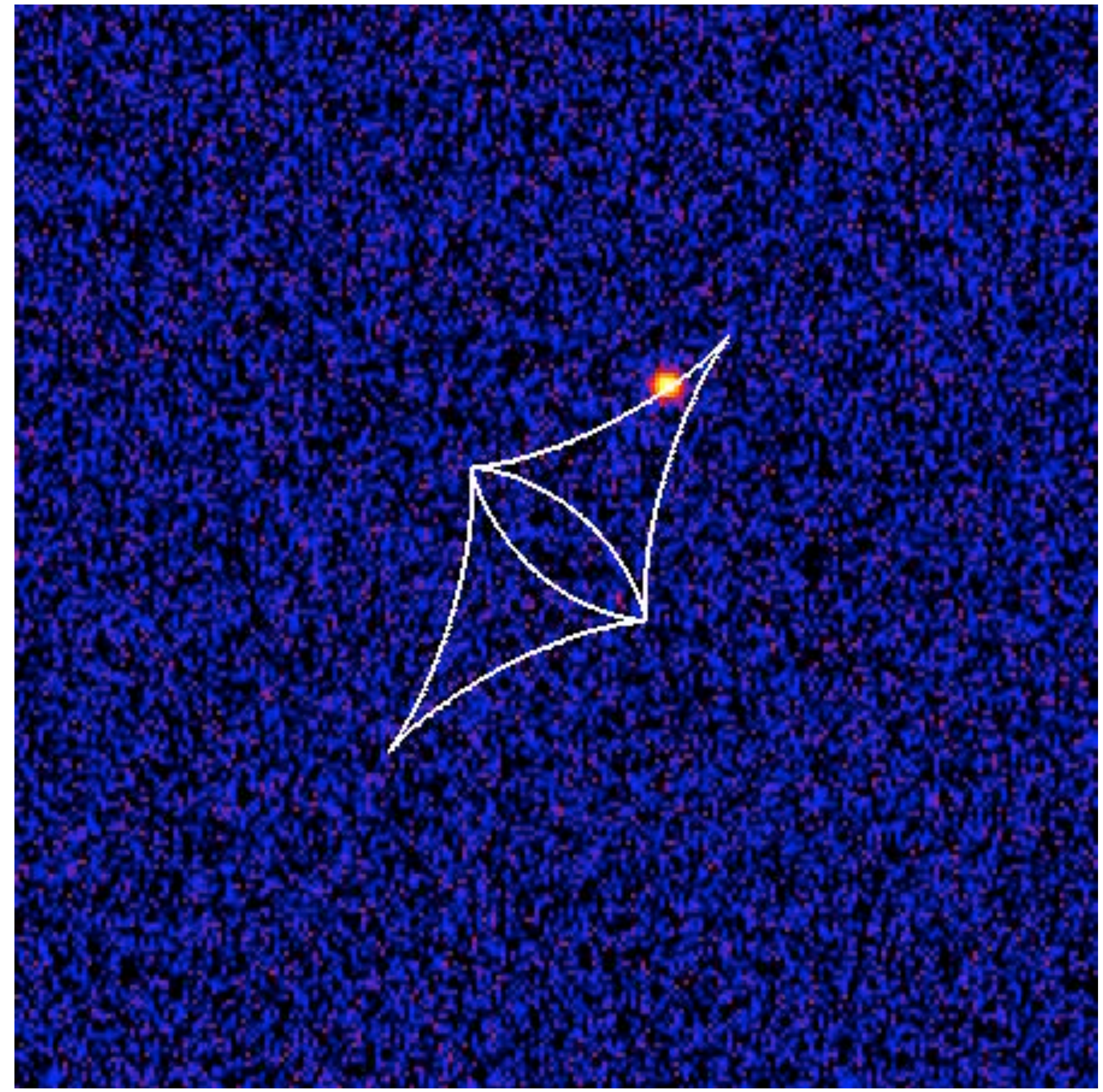


image plane
(critical curves)



source plane
(caustics)

Example: extended source (galaxy)

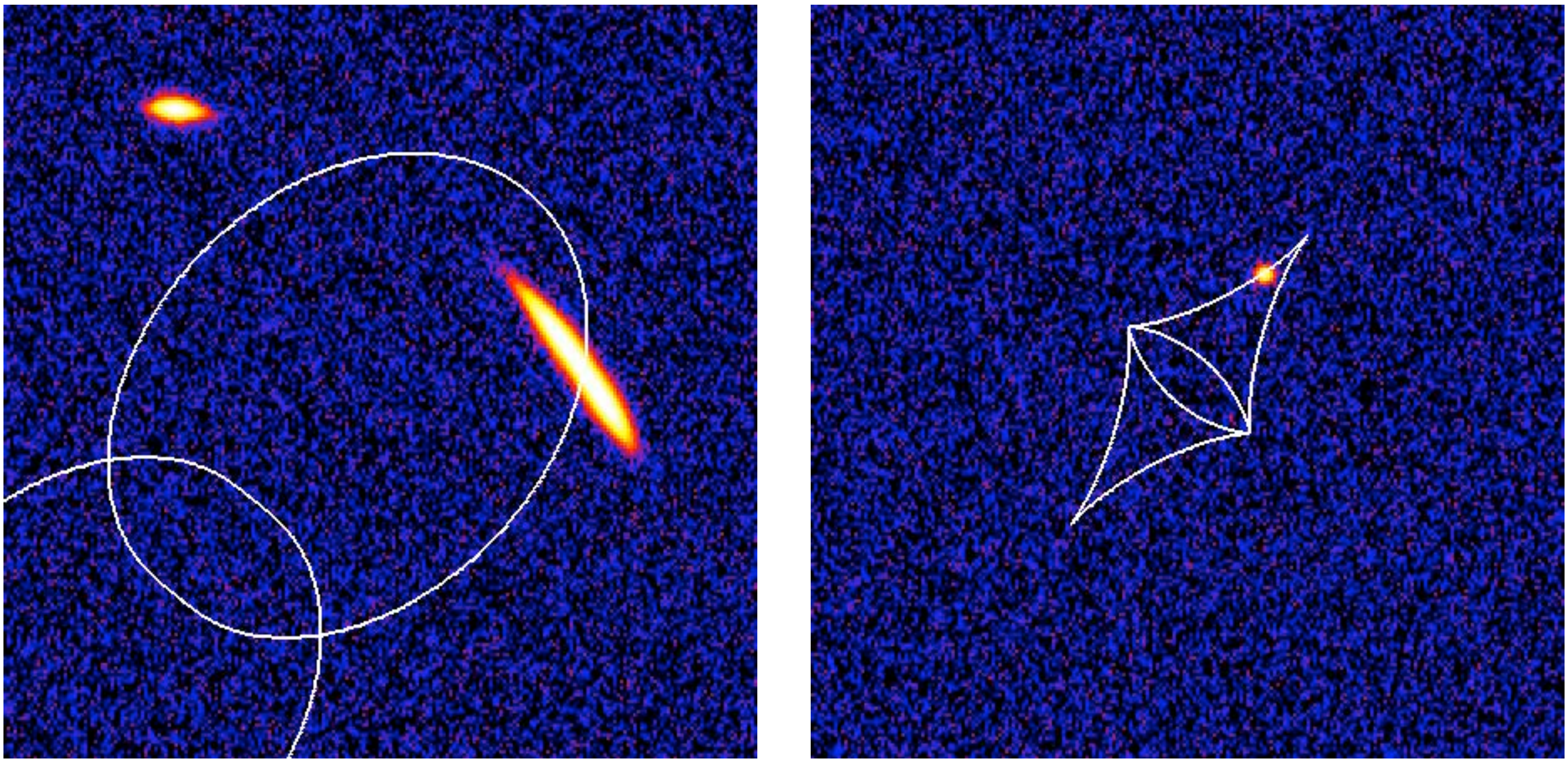


image plane
(critical curves)

source plane
(caustics)

simulated by *glafic*

Example: extended source (galaxy)

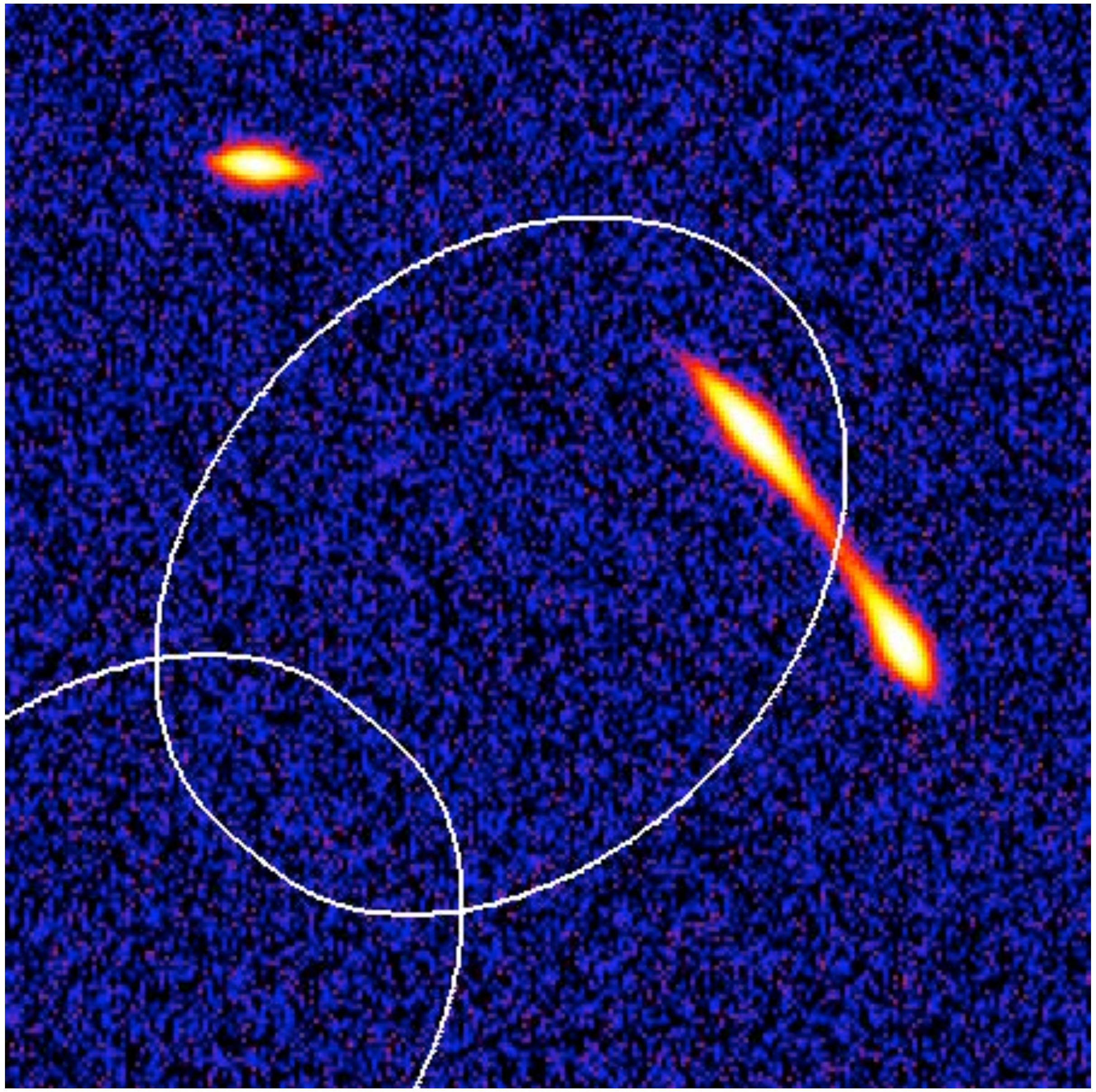
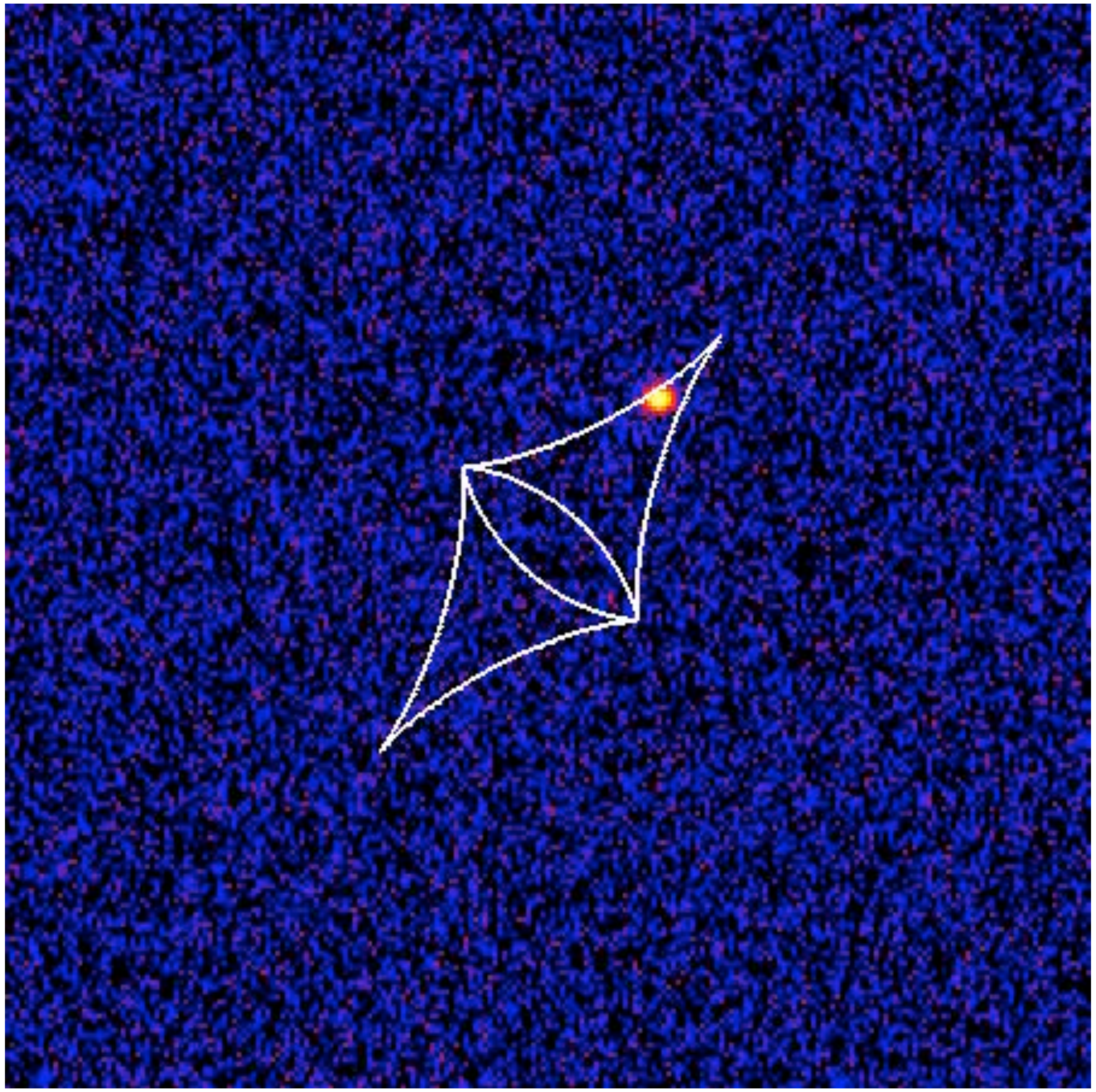


image plane
(critical curves)



source plane
(caustics)

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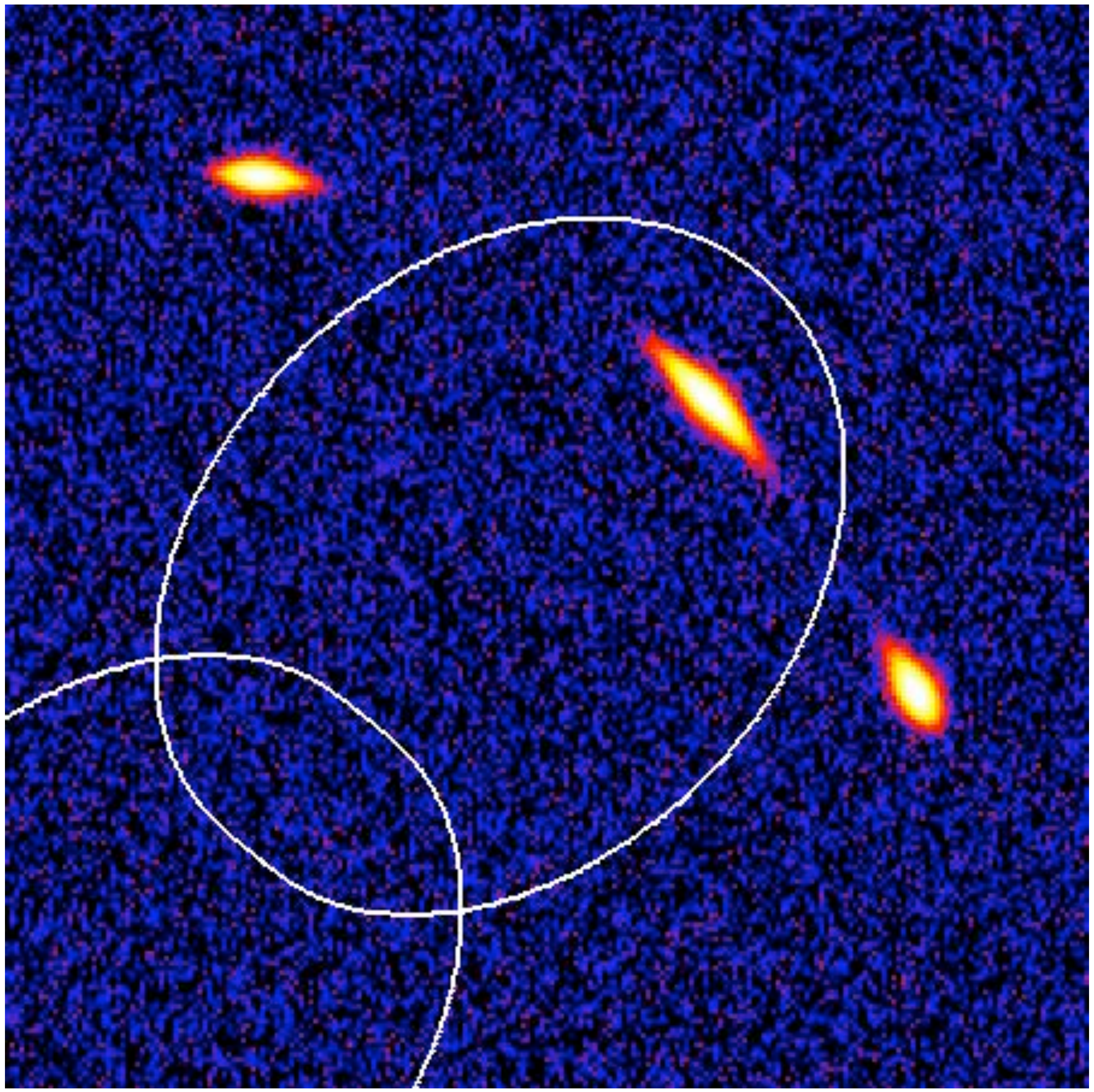

image plane
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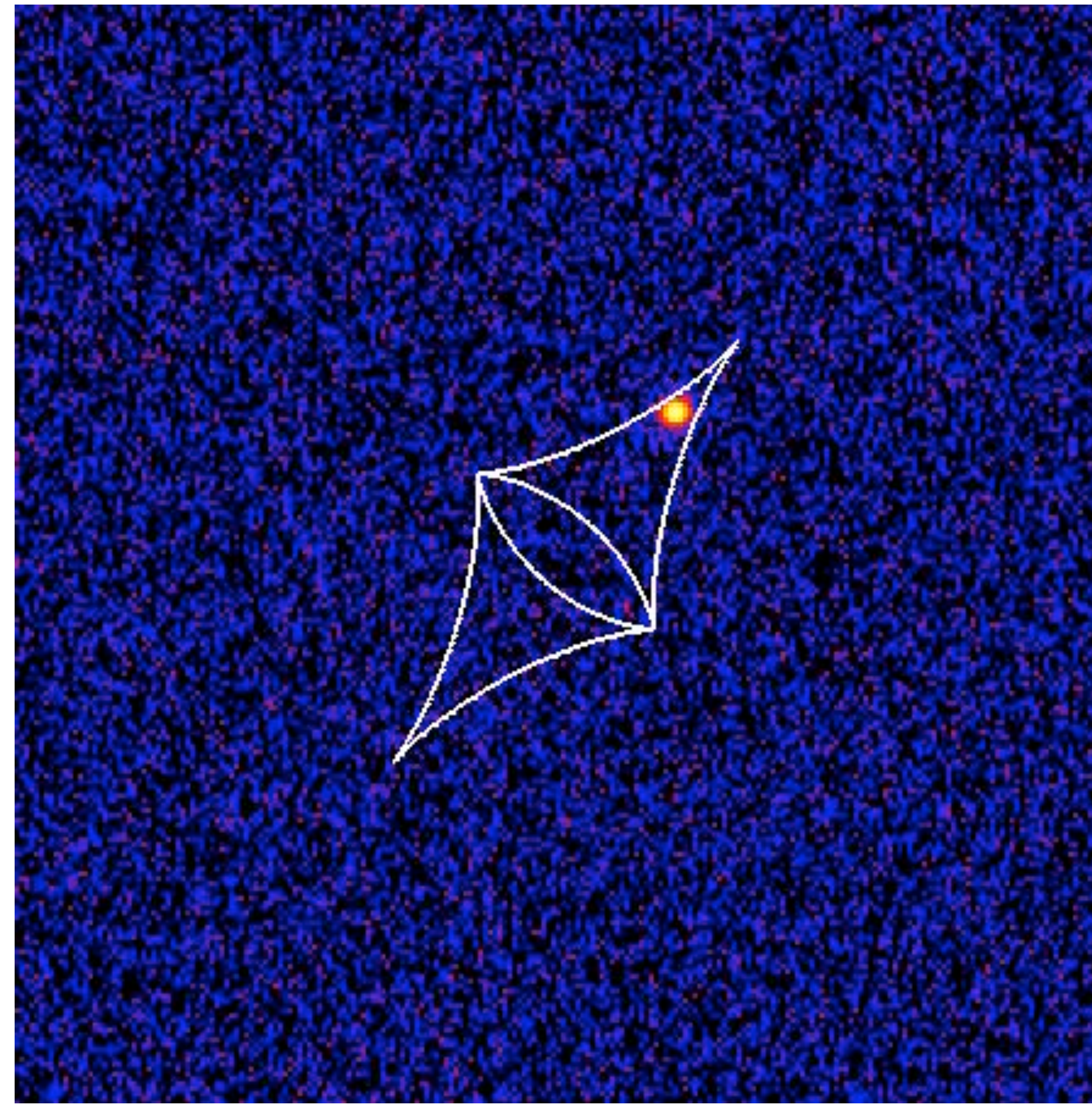
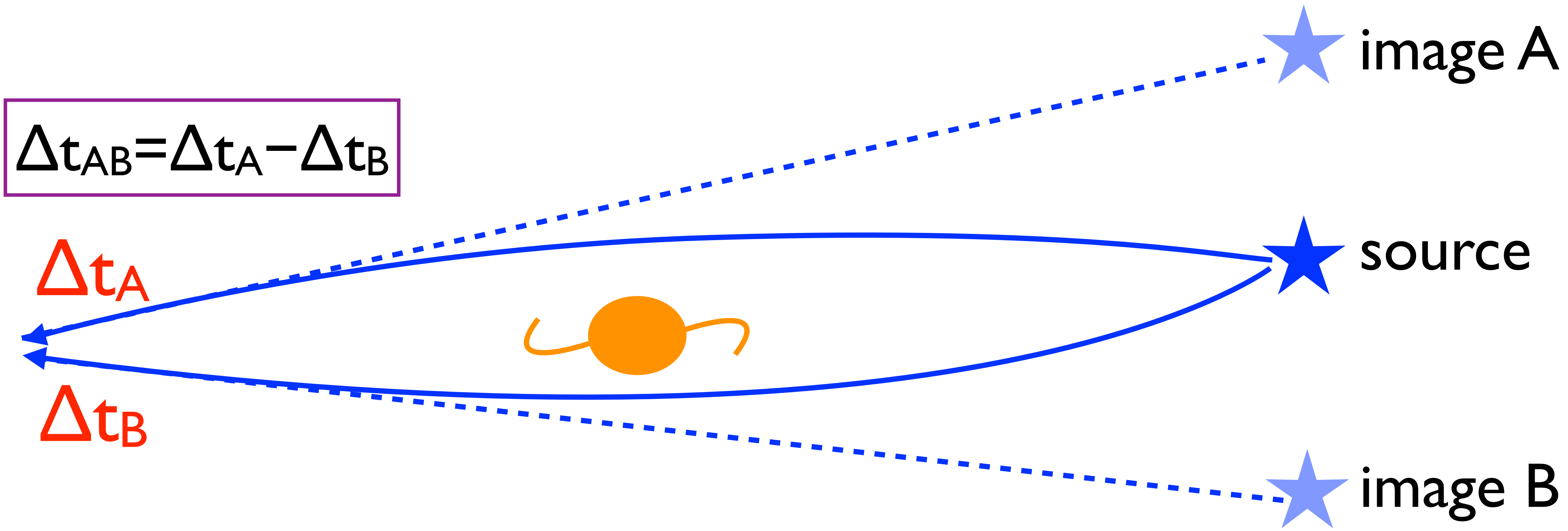


source plane
(caustics)

Time delay

- different light paths have different travel time
- travel time difference can be measured for time-variable sources (e.g., quasar)



Deriving time delay: outline (I)

- light travels null geodesic $ds^2=0$

$$c\,dt = \left(1 - \frac{2\phi}{c^2}\right) a\,dl \qquad dl \equiv \sqrt{\gamma_{ij}dx^i dx^j}$$

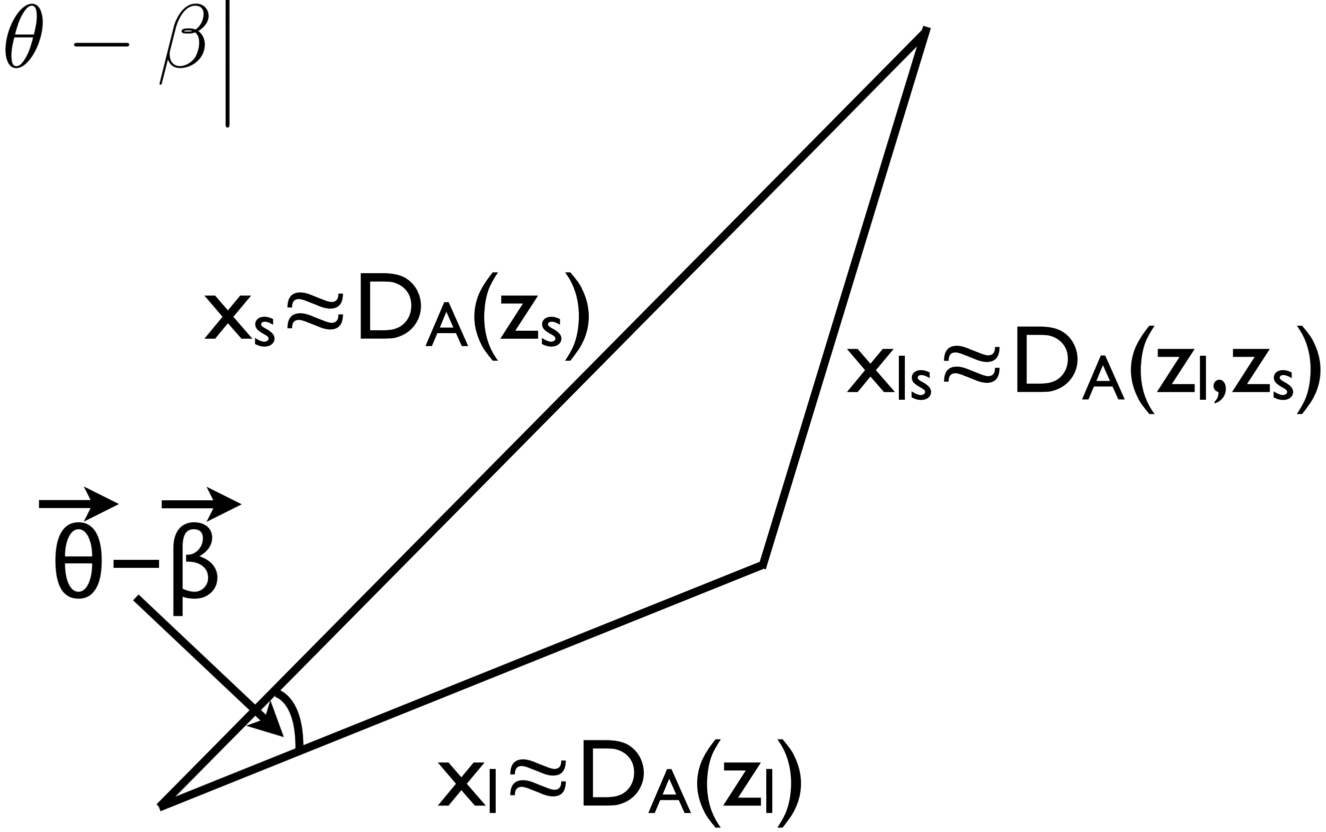
$$c\Delta t_{\rm lens} = \underbrace{\Delta x_{\rm lens}}_{\text{geometrical time delay}} \underbrace{- \frac{2}{c^2}\int \phi\, a\, dl}_{\text{gravitational (Shapiro) time delay}}$$

Deriving time delay: outline (II)

- geometrical delay

$$\cos\left|\vec{\theta}-\vec{\beta}\right| \simeq 1-\frac{1}{2}\left|\vec{\theta}-\vec{\beta}\right|^2 \simeq 1-\frac{\Delta x_{\rm lens} x_{ls}}{x_l x_s}$$

$$\rightarrow\ \Delta x_{\rm lens} \simeq \frac{D_A(z_l)D_A(z_s)}{2D_A(z_l,z_s)}\left|\vec{\theta}-\vec{\beta}\right|^2$$



Deriving time delay: outline (III)

- gravitational time delay

from the definition of lens potential ψ

$$\frac{2}{c^2}\int \phi\, a\, dl \simeq \frac{f_K(\chi_l) f_K(\chi_s)}{f_K(\chi_s - \chi_l)} a_l \psi = \frac{D_A(z_l) D_A(z_s)}{D_A(z_l, z_s)} \psi$$

Deriving time delay: outline (IV)

- cosmological time dilation

$$\Delta t_{\rm obs} = (1+z_l)\Delta t_{\rm lens}$$

- total observed time delay is given by

$$c\Delta t_{\rm obs} = (1+z_l)\frac{D_A(z_l)D_A(z_s)}{D_A(z_l,z_s)}\left[\frac{1}{2}\left|\vec{\theta}-\vec{\beta}\right|^2-\psi\right]$$

Time delay and H_0

- time delay is known to provide a unique probe of the *absolute* distance scale, H_0

$$\underbrace{c\Delta t_{\rm obs}}_{\substack{\text{observe}\\\text{(typically a}\\\text{few months)}}} = (1+z_l)\underbrace{\frac{D_A(z_l)D_A(z_s)}{D_A(z_l,z_s)}}_{\substack{\text{constraint on}\\\text{the distance ratio}\\\propto H_0^{-1}}}\underbrace{\left[\frac{1}{2}\left|\vec{\theta}-\vec{\beta}\right|^2-\psi\right]}_{\substack{\text{observe }(\theta)\text{ +}\\\text{mass modeling}}}$$

Mass-sheet degeneracy (I)

- consider the following transform

$$\kappa(\vec{\theta}) \to \lambda\kappa(\vec{\theta}) + (1-\lambda)$$

observable

$$\boxed{\vec{\theta} \to \vec{\theta}}$$

- then other quantities transform as

$$\psi(\vec{\theta}) \to \lambda\psi(\vec{\theta}) + (1-\lambda)\frac{\theta^2}{2}$$

$$\boxed{g_a \to g_a}$$

$$\vec{\alpha}(\vec{\theta}) \to \lambda\vec{\alpha}(\vec{\theta}) + (1-\lambda)\vec{\theta}$$

$$\mu \to \lambda^{-2}\mu$$

$$\vec{\beta} \to \lambda\vec{\beta}$$

$$\boxed{\mu_i/\mu_j \to \mu_i/\mu_j}$$

$$\gamma_a \to \lambda\gamma_a$$

Mass-sheet degeneracy (II)

- on the other hand, time delays transform

$$\Delta t_{ij} \to \lambda \Delta t_{ij} \quad (\mathrm{fixed\ H_0})$$

or

$$\Delta t_{ij} \to \Delta t_{ij} \quad (H_0 \to \lambda H_0)$$

$\boxed{\text{mass-sheet degeneracy is one of the most important systematics on } H_0 \text{ from time delays!}}$

Strong vs weak lensing

- strong lensing
  - observed for individual sources
  - $\kappa \gtrsim 1$ ($\Sigma \gtrsim \Sigma_{cr}$), near critical curves/caustics
  - multiple images, high elongation/magnification
- weak lensing
  - observed for ensemble of sources
  - $\kappa \ll 1$ ($\Sigma \ll \Sigma_{cr}$), far from critical curves/caustics
  - no multiple image, tiny elongation/magnification

Summary

- lens equation is a key equation for various lensing analysis
- it is essentially a mapping between source and image, and is derived from geodesic equation
- explained several key concepts: convergence, shear, magnification, critical curves, caustics, time delays, ...

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